THEORY OF NUMBERS, Math 115 B
Homework 2

1. Prove that for the area $S(M)$ of a simple lattice polygon M the following inequality holds:

$$S(M) \geq G - L/2 - 1$$

where G denotes the total number of lattice nodes lying inside M (i.e. $G = i + b$), L denotes the perimeter of the polygon (i.e. the length of its boundary). Can you think of a case when equality holds?

2. Construct a set C in the plane that is convex, symmetric about 0 and has area 4, but contains no nonzero integral point.

3. Construct a set C that is convex and has infinite area but contains no integral point.

4. Can you write every non-negative integer as a sum of three squares?