THEORY OF NUMBERS, Math 115 B
Homework

1. Prove that any triangulation of a simple polygon with n vertices has $n-3$ diagonals and $n-2$ triangles.

2. Prove that any triangulation of a simple polygon has an ear: i.e. a triangle that shares a single edge

3. Suppose a, b are relatively prime integers. Prove that there are no lattice points in the interior of the segment going from $(0,0)$ to (a,b).

4. Determine the number of lattice points in the segments $(12,24)-(17,20)$ $(3/4,8/7)-(3,11/7)$

5. Find out the number of lattice points inside the polygon $P = v_0, v_1, ...v_5$ with vertices $v_0 = (0,0)$, $v_1 = (1,1)$, $v_2 = (2,4)$, $v_3 = (3,9)$, $v_4 = (4,16)$, $v_5 = (5,25)$.

6. Using straighline segments, draw a nonconvex figure with symmetry about $(0,0)$, area more than 4, and no lattice points except $(0,0)$ inside it or on its boundary.

7. Find a tetrahedron with integer coordinates for its vertices but no lattice points on its boundary or interior, whose volume is more than $1/6$. This makes a generalization of Pick's theorem impossible.

8. A convex set Q contains three non-collinear points A, B, C. Prove that then Q must contain every point of the triangle ABC.

9. (application of Minkowski theorem) Using Minkowski's geometric thinking prove the following number theory statement Given any real number α and an integer $t > 0$, there exist integers p, q no both zero such that $|q/p - \alpha| \leq 1/t$. (HINT: take the right M-set and apply Minkowski's theorem to it).