Math 128C (De Loera) **Homework 3**
Due date: May 23, 2018

INSTRUCTIONS

All homeworks will have many problems, both theoretical and practical. Programming exercises need to be submitted via CANVAS.

Theory part must be scanned or, even better, please use LateX! Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. Section 5.10 #2, 5, 7, 8
2. Section 5.11 (Stiffness) # 9, 10, 11, 14 and Discussion #1,2
3. COMPUTER PROJECT 1: Some famous systems of ODEs: Lotka-Volterra

A classical model in ecology is the Lotka-Volterra predator-prey mode. Consider an ecosystem consisting of rabbits that have infinite supply of food and foxes who prey on the rabbits for their food. This is modeled by:

$$\frac{dr}{dt} = 2r - \alpha r f,\ r(0) = r_0$$
$$\frac{df}{dt} = -f + \alpha r f,\ f(0) = f_0$$

where t is time, $r(t)$ is the number of rabbits and $f(t)$ the number of foxes, and α is a positive constant. The system cannot be solved explicitly! Thus numerical analysis comes to the rescue!! You are a mathematician helping biologists, explain:

a) How the behavior of the system depends on α. Explain what happens for $\alpha = 0, 0.01, 2, 5, 10$ Your explanation should be expressed in terms of the animals (e.g., foxes population increases Show experiments that support justify your reasoning. What is the meaning of a negative α?

b) Next, play with several different values of the initial conditions. Setting $\alpha = 0.01$ explore what happens when changing the initial conditions, e.g., few rabbits vs lots of foxes, equal populations, etc. Try the 6 combinations below and for each combination present two plots: One for the functions r, f as functions of t and another a phase plot with r as one axis and f as the other. Let t grow, is the behavior periodic?

$r_0 = 3000, f_0 = 50,$

$r_0 = 300, f_0 = 150,$

$r_0 = 150, f_0 = 1000,$

$r_0 = 102, f_0 = 198,$

$r_0 = 100, f_0 = 200$

c) In nature the number of rabbits does not grow indefinitely. So it makes sense to modify the equations to account for that:

$$\frac{dr}{dt} = 2(1 - \frac{r}{R})r - \alpha r f,\ r(0) = r_0$$
$$\frac{df}{dt} = -f + \alpha r f,\ f(0) = f_0$$

Explain why in this new model the number of rabbits now cannot exceed R. For $\alpha = 0.01, R = 400$, compare the behavior of this new model to the previous one. Use the initial conditions $r_0 = 300, f_0 = 150$, to compare the two models in 4 different plots:

- number of foxes and number of rabbits versus time for original model
- number of foxes and number of rabbits versus time for modified model
- number of foxes versus number of rabbits for original model
- number of foxes versus number of rabbits for modified model

4. COMPUTER PROJECT 2:

5. Section 11.1 #2, 6, 9.

6. Section 11.2 #2, 4 a,c., 7

7. Section 11.3 #2, 4

8. Section 11.4 #2,4,5.

9. COMPUTER PROJECT : Now we consider the computation of boundary value problems:

   a) Consider the code I gave you in class for the non-linear shooting method. Rewrite that code to replace Newton's method with the Secant Method. Use $t_0 = (\beta - \alpha)/(b - a)$ and $t_1 = t_0 + (\beta - y(b, t_0))/(b - a)$. Repeat exercises 4a,c using the secant version and compare what you saw using the Newton method.

   b) Consider the boundary Value Problem

   $$y'' = y^2 - 1, \quad y(0) = 0, \quad y(1) = 1$$

   . This problem can be solved in at least 2 ways: Using the shooting method and finite differences. Modifying the code I gave you in class, compute the two solutions and make plots, presented in a single figure, for both methods of solution. Discuss the differences.