Math 128C (De Loera) **Homework 3**
Due date: June 8th, 2018

INSTRUCTIONS

All homeworks will have many problems, both theoretical and practical. Programming exercises need to be submitted via CANVAS.

Theory part must be scanned or, even better, please use LateX! Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. 12.1 # 2,3a,c, 6
2. 12.2 #2,4,5,18
3. 12.3 #1,3,4
4. 12.4 # 1,2,4.
5. COMPUTER PROJECT 1:

Consider the following problem:

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial^2 u(x,t)}{\partial x^2}, \quad 0 \le x \le 1, \quad t > 0.$$

The ends of the rod are kept in contact with blocks of melting ice (that is, $u(0,t) = u(1,t) = 0$ for all t). The initial temperature distribution is

$$\begin{aligned} u &= 2x & 0 \le x \le 0.5 \\ u &= 2(1-x) & 0.5 \le x \le 1. \end{aligned}$$

(A) Verify that

$$u = \frac{8}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (\sin \frac{n\pi}{2})(\sin n\pi x) \exp(-n^2\pi^2 t)$$

is the analytical solution to the above problem.

(B) Using Newton's forward difference method solve the above with

(1) $h = \frac{1}{10}, \quad k = \frac{1}{1000}$.

(2) $h = \frac{1}{10}, \quad k = \frac{5}{1000}$.

(3) $h = \frac{1}{10}, \quad k = \frac{1}{100}$.

Plot your solution at $x = 0.0, 0.1, 0.2, \cdots, 1.0$ and compare it to the analytic solution.

(C) Do the same as in part (B) except use Newton's Backwards method.

(D) Do the same as in part (B) except use the Crank-Nicolson method.

(E) Compare and comment on your solutions.

6. COMPUTER PROJECT 2: Modify the code provided in class to solve the following problems:

Solve the elliptic partial differential equations with Dirichlet boundary conditions by the Finite Element Method with $h = k = 0.1$. Plot the solution.

(a) $$\begin{cases} \Delta u + \sin \pi x y = (x^2 + y^2)u \\ u(x,0) = 0 \text{ for } 0 \le x \le 1 \\ u(x,1) = 0 \text{ for } 0 \le x \le 1 \\ u(0,y) = 0 \text{ for } 0 \le y \le 1 \\ u(1,y) = 0 \text{ for } 0 \le y \le 1 \end{cases}$$

(b) $$\begin{cases} \Delta u + (\sin \pi x y)u = e^{2xy} \\ u(x,0) = 0 \text{ for } 0 \le x \le 1 \\ u(x,1) = 0 \text{ for } 0 \le x \le 1 \\ u(0,y) = 0 \text{ for } 0 \le y \le 1 \\ u(1,y) = 0 \text{ for } 0 \le y \le 1 \end{cases}$$