Math 128C (De Loera)   **Homework 1 (One Step Methods Chapter 5)**
Due date: April 18, 2014

INSTRUCTIONS
All homeworks will have many problems, both theoretical and practical. Programming exercises need to be submitted via CANVAS.

Theory part must be scanned or even better please use LateX. Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. Find a problem that interests you and has a nice ODE model, e.g., in Physics, Chemistry, social sciences, etc. Describe the model, use MATLAB ODE solvers to find solutions. Does your ODE have a close analytic formula solution?

2. Solve the following initial value problems analytically and give an argument as to why they have a unique solution:

   - $y' = t^3, y(0) = 0$
   - $y' = ay + b, y(0) = y_0$, where a, b are given scalars.

3. 5.1 #4,6ac.

4. 5.2 #2, 17.

5. 5.3 # 2ab, 4ab

6. 5.4 # 2ab, 6ab, 29,32.

7. COMPUTER PROJECT 1: Write your own program to solve the problems of the form $y' = f(t, y), y(t_0) = y_0$ using two different methods: Euler's method and the modified Euler's method (page 286 of the book). Your program should work with general functions $f(t, y)$ stepsize h and any interval $[a, b]$.

   (a) Using your program solve the following IVP:

   $$y' = -4t^3 y^2; \quad y(-a) = \frac{1}{(a^4 + 1)}; \quad t \in [-a, a]$$

   You may choose $a = 5$ or $a = 10$.
   - show that $y(t) = \frac{1}{t^4+1}$ is the exact solution of the IVP for $a = 0$.
   - Find the values of $y(t)$ after using stepsizes $h = 10^{-3}, 10^{-4}, 10^{-5}$. For each value of h, graph the approximate solution $y(t, h)$ for t going from $[-a, a]$.

   (b) Analyze your output and supply written comments on it. Assuming the true answer is NOT known, how would you **estimate** the accuracy of Euler's method for this problem? Use different values of h. Also use your error formulas to obtain an error bound for Euler's method (not for the modified Euler!). Compare this bound to the error in your calculations.

(c) Assume that the true solution $y(t)$ and the approximate solution $y(t, h)$ obtained from Euler's method with step size h satisfy a relation of the form $y(t) = y(t, h) + m_1 h + m_2 h^2 + O(h^3)$. Use Richardson's extrapolation process to show how $O(h^2)$ and $O(h^3)$ approximations to the true solution can be obtained using the results from Euler's method for step sizes h, $0.1h$, and $0.01h$. Give specific formulas for these improved approximations.

(d) Using the results obtained in Part (a), compute the improved (Richardson) approximations to the numerical solution of the stated initial value problem. Estimate the error at $t = 0$ for each of your new $y^*(t, h)$. Compare this to the results of Modified Euler's method from Part (a).