Combinatorics, Math 145
Homework one, Due April 7

1. Problems from section 1.8: 8, 24, 31, 34.

2. You are on vacation and wish to send your n most favorite professors 2 different postcards each. There are k kinds of postcards. How many different ways are there to do this? (note: it is ok if two professors get the same card, after all they are from different departments).

3. Draw n lines in the plane in such way that no two are parallel and no three intersect in a common point. Prove that the plane is divided into $\frac{n(n+1)}{2} + 1$ regions.

4. Among the integer numbers $1, 2, \ldots 10^{10}$, are there more of those containing 9 in their decimal notation or those with no 9?

5. How many permutations of $1, 2, \ldots n$ have a single cycle?

6. Device a way to compute the order of a permutation. Test your algorithm with the permutation $[2, 3, 1, 5, 4, 7, 8, 9, 6]$.

7. Consider the numbers $1, 2, \ldots, 1000$, show that among any 501 of them there are two numbers such that one divides the other one. Hint: think of a way to use the pigeonhole principle.