Combinatorics, Math 145
Homework one, Due April 21

1. Problems 2.5.4, 2.5.5, 2.5.8.

2. How many terms are there in the sum on the right-hand side of the formula for $(x_1 + x_2 + \ldots + x_m)^n$ in the multinomial theorem.

3. Let p be a prime and let n, k be natural numbers (a) Prove that for $k < p$, $\binom{p}{k}$ is divisible by p. (b) Prove that $\binom{n}{p}$ is divisible by p if and only if $\lfloor n/p \rfloor$ is divisible by p.

4. Give and prove a formula for $\sum_{i=1}^{n} i^3$.

5. In rectangular grid city, a student walks from home to school which is located 10 blocks east and 14 blooks north from her home. She always takes a shortest walk of 24 blocks. How many different walks are possible? Suppose that 4 blocks east and 5 blocks north of her home live her best friend, whom she picks up each day on her way to school. Now how many different walks are possible?

6. Determine the number of permutations of $\{1, 2, \ldots, 8\}$ in which no even integer is in its natural position.

7. Prove that in group of $n > 1$ people there are two who have the same number of acquaintances in the group.

8. There are 100 people at a party. Each person has an even number of (possibly zero) acquaintances. Prove that there are three people at the party with the same number of acquaintances.