Combinatorics, Math 145
Homework six, due May 24th

1. Using the recursive formula (deletion-contraction) calculate the number of trees for $K_{3,3}$.

2. Use Kruskal's algorithm to find the optimal tree connecting the following big cities in the world: (L) London, (MC) Mexico city, New York, Paris, Beijing and Tokyo. Distances in miles or kilometers can be found at `http://www.geobytes.com/CityDistanceTool.htm`. What is the shortest Traveling salesman tour?

3. 8.5.1, 8.5.5, 8.5.9

4. 9.2.2, 9.2.3, 9.2.8.

   Hint for 9.2.3: In the spirit of Kruskal's theorem proof: Suppose edges are in order of cost $e1, e_2, \ldots, e_{n-1}$etc. Suppose not unique optimal tree, call K the tree constructed by Kruskal's algorithm, and T an optimal tree with the **largest** e_k first edge not present in K.

   Let S be the partial tree constructed by Kruskal before e_k is added. e_k forms a cycle in K, inside it there is e^* with one end in S and the other not in S. Prove that $U - e^* + e_k$ is another optimal tree which gives a contradiction.

5. 10.1.2, 10.3.1.