

Math 146
Homework 5 (last homework!)

1. Show that the number of partitions of n with at most two parts is $\lfloor \frac{n}{2} \rfloor$.
2. The number of partitions of n in which each part is 1 or 2 is equal to the number of partitions of $n + 3$ which have exactly two distinct parts.
3. Find the least value of n for which $p(n) > 1000$.
4. Prove that the following quantities are all given by the sequence of Catalan numbers:
 - a) Given $2n$ (equally spaced) points be chosen in a circle, the number of ways to join these points in pairs, so that the resulting n line segments do not intersect.
 - b) The number of binary trees with n nodes.
 - c) Given n numbers being multiplied $a_1 a_2 \dots a_n$, the number of ways to put parenthesis in the multiplication in order to indicate the order of operations. e.g. for 3 numbers some possibilities are $(a_1(a_2 a_3))$, $(a_1(a_2 a_3))$, etc.
 - d) The number of $2 \times n$ arrays (x_{ij}) that can be made with the numbers 1 to $2n$ so that $x_{i1} < x_{i2} < \dots < x_{in}$ and $x_{1j} < x_{2j}$. for $i = 1, 2$ and $j = 1 \dots n$.