

This is an introduction to MAPLE which is a program for doing mathematics with computers (a computer algebra system). We will focus on permutations, groups, and discrete mathematics

We work all the time with permutations  $S_n$  for some  $n$ . A PERMUTATION inside MAPLE can be represented in two ways:

1) As permutation list (1  $\rightarrow$  3, 2  $\rightarrow$  1, 3  $\rightarrow$  2) which is encoded by a list [3, 1, 2].

2) As the product of disjoint cycles: (1,2) (3,2,4) is encoded in a list of lists [[1,2],[3,2,4]].

We can construct many many groups using permutations. In fact we can construct ALL finite groups this way! Here is an example:

The cheap\_rubik box is a toy like the famous Rubik's cube. Instead of a 3x3 cube is a 3x2 box.

The point is that once we label the faces of cheap\_rubik box we can express the mechanical moves by a series of permutations of 18 symbols of letters!! See picture for a description of the toy

The subgroup of  $S_{18}$  generated by these permutations is precisely the group of operations for the cheap\_rubik box. Now here is a practical problem. I give you a configuration that could in principle be a valid cheap\_rubik configuration. How can we decide whether there are moves that really achieve it? Take for example [10, 6, 9, 11, 5, 8, 12, 4, 7, 3, 2, 1, 13, 14, 15, 16, 17, 18]:

Let's begin our computer exploration:

Since we are beginners we can just start by asking for help:

```
> help(group);
```

You can use this as a reference for all kinds of commands inside MAPLE. Now we call the library of commands that deals with groups and permutations.

```
> with(group);
```

Once we have all the commands available we can define inside maple the group of moves: These are certain "basic" permutations from which any other configuration

can be obtained. They are the generators of the group!!

```
> moves_rubik:={a=[[1,3,9,7],[2,6,8,4]],
  b=[[10,12,18,16],[11,15,17,13]],
> c=[[1,12],[3,10],[2,11]],d=[[3,18],[9,12],[
  6,15]],
  e=[[9,16],[7,18],[8,17]],f=[[7,10],[1,16],[
  4,13]]};
```

```
> cheaprubik:=permgrou(18,moves_rubik);
```

Then we can ask questions about the group: How many elements? Is the group Abelian?, etc:

```
> grouporder(cheaprubik);
```

Another possibility is to ask whether an specific permutation belongs to the group CHEAPRUBIK.

```
> groupmember([[1,10,3,9,7,12],[2,6,8,4,11]],
  cheaprubik);
```

Here is the same question for a different permutation!

```
> groupmember([[1,10,11]],cheaprubik);
```

Here is a way of multiplying to permutation as long as they are in cycle notation

```
> mulperms([[2,3,4],[1,6]], [[4,6]]);
```

Of course, it then becomes necessary to transfer between the cycle notation and the list notation. Here is how to do it:

```
> convert([[2, 8, 13, 11, 4, 6, 15, 17], [3,
  16], [7, 9], [10, 18]], 'permlist', 18);
```

```
> convert([10, 6, 9, 11, 5, 8, 12, 4, 7, 3,
  2, 1, 13, 14, 15, 16, 17, 18], 'disjcytc');
```

Now suppose you wish to decide what are the cosets of a certain subgroup ( like in the proof of Lagrange theorem!). First you input the

[ group and the candidate subgroup:

```
[ > pg1:=permgrou(4, {[[1,2,3]], [[1,3,2]], [[1,3,4]], [[1,4,3]], [[1,2,4]], [[1,4,2]], [[2,3,4]], [[2,4,3]]});
```

```
[ > grouporder(pg1);
```

```
[ pg2:=permgrou(4, {[[1,2], [3,4]]});
```

But we wish first to be sure pg2 is a subgroup! Else the calculation makes no sense.

```
[ > issubgroup(pg2, pg1);
```

Finally we ask for the list of cosets to be computed. Did we expect that answer?

```
[ > cosets(pg1, pg2);
```

```
[ >
```