Math and Computers, Math 165
Programming Project, Due April 8, 2006

1. Prove that every univariate polynomial with complex coefficients and degree m has at most m distinct roots.
2. Use MAPLE to find all the real roots of the polynomial $3x^5 - 25x^3 + 60x - 20$.
3. Is the polynomial $x^2 - 4$ in the ideal generated by the polynomials $x^3 + x^2 - 4x - 4, x^3 - x^2 - 4x + 4, -2x^2 - x - 2$.
4. Given univariate polynomials $f_1, \ldots, f_s$. Prove that the set of common roots of these polynomials (the variety) is empty if and only if their GCD is 1.
5. Use MAPLE to find the square-free part of the polynomial $x^{11} - x^{10} + 2x^8 - 4x^7 + 3x^5 - 3x^4 + x^3 + 3x^2 - x - 1$.
6. **FIRST PROJECT ASSIGNMENT:**
   - Read the article "the death of proof" in Scientific American October 1993 (available from this course's web page). Write an essay, of no more than two pages of length, expressing your opinion as to why computer-based proofs are acceptable or not acceptable in modern mathematics. Try to answer the question "why do we need proofs in mathematics?" Try to give some justification of your statements.
   - Carry on the following experiment with MAPLE: Learn how to generate random univariate polynomials with fixed number of terms using randpoly. For many such polynomials use the *solve* command to find its roots and count the number of real roots you have,tabulate this against the degree and the number of terms. Make sure your polynomials take on a wide range of coefficients. Can you extract some kind of rule as to what is the largest possible number of real roots?
   - Write a MAPLE program that implements the classical extended Euclidean algorithm over $Q[x]$. Experiment with 100 pairs of random polynomials with coefficients over $\mathbb{Z}$ (calculations are done over the rationals). Do you notice something on the coefficients of r_i rem r_{i-1}? How often are the polynomials relatively prime?