1. Applying the multivariate division algorithm we learned in class compute the remainder of \( f = xy^2z^2 + xy - yz \) with respect to the polynomials \( \{x - y^2, y - z^3, z^2 - 1\} \) given in that order. Repeat the exercise with the order of the set permuted cyclically. Check your hand-made answer with MAPLE.

2. Let \( I = \langle x^6, x^2y^3, xy^7 \rangle \). In the plane, plot the set of all exponent vectors \((m, n)\) of monomials \(x^my^n\) appearing in \(I\). If we apply the division algorithm to a polynomial \(f\) using the generators of \(I\), what terms can appear in the remainder? Show it in a nice picture!

3. A basis for a monomial ideal \( \{x^{a_1}, x^{a_2}, \ldots, x^{a_s}\} \) is said to be minimal if no \(x^{a_i}\) in the basis divides another \(x^{a_j}\). Prove that a monomial ideal has a unique minimal basis.

4. Show that for each \(n\) there exist a monomial ideal \(I \subset K[x, y]\) such that every basis of \(I\) has at least \(n\) elements.

5. Compute the S-pairs of the following polynomials using the lexicographic monomial order.

- \(f = 4x^2z - 7y^2, \ g = xy^z^2 + 3xz^4\).
- \(f = x^4y - z^2, \ g = 3xz^2 - y\)
- \(f = xy + z^3, \ g = z^2 - 3z\).