1. Calculate the remainder of \( x^3 - x^2y - x^2z + x \) by \( f_1 = x^2y - z \), and \( f_2 = xy - 1 \). Compute the remainder of \( f \) on division by \( (f_1, f_2) \).

2. Using Lagrange multipliers find the point(s) on the surface \( x^4 + y^2 + z^2 - 1 = 0 \) closest to the point \((1, 1, 1)\). HINT: You have to solve a system of equations!

3. Using Gröbner bases find all critical points of the polynomial function (check your calculus book for the definition).

\[
f(x, y) = (x^2 + y^2 - 4)(x^2 + y^2 - 1) + (x - \frac{3}{2})^2 + (y - \frac{3}{2})^2
\]

4. What is the Buchberger criterion, show an example.

5. Find all solutions of the system of equations \( x^2 + y^2 + z^2 - 1 = 0, x^2 + y^2 + z^2 - 2x = 0, 2x - 3y - z = 0 \).

6. State the Hilbert Nullstellensatz.