Math 22A (De Loera) **Name:**
Final Exam **Student ID#**
June 12, 2004

INSTRUCTIONS

(1) READ INSTRUCTIONS CAREFULLY!

(2) DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO.

(3) FILL IN THE INFORMATION ON THIS PAGE (your name, etc) NOW!!

(4) SHOW YOUR WORK on every problem. Justify EACH step or conclusion!! Correct answers with no support work or explanations will not receive full credit.

(5) PLEASE WRITE LEGIBLY. Be organized and use the notation appropiately.

(6) NO EXTRA ASSISTANCE ALLOWED. Assistance from classmates, notes, books or calculators is prohibited. You should only have a pencil and eraser on your desk.

(7) USE THE BACK SIDE OF THE PAPER if you need extra space.

#	Student's Score	Maximum possible Score
1		6
2		5
3		6
4		3
Total points		20

1. (6 points) Find an *orthogonal* basis for the solution space of the system $Ax = 0$ where A is given by the matrix below. What is the value for $rank(A)$?

$$\begin{bmatrix} 1 & 0 & -1 & 2 \\ 2 & 1 & -2 & 2 \\ 0 & 1 & -2 & 4 \end{bmatrix}$$

2. (5 points)

Of the following 3 transformations from R^3 into R^3, decide which one is (the only) linear one (explain why). Then consider that linear transformation you found and determine whether (a) it is an onto or (b) a one-to-one linear transformation. Find the dimension of the range and the kernel.

$a) L_1(x, y, z) = (x + 2y + z, x + y, 2y + z).$

$b) L_2(x, y, z) = (x^3, y, z).$

$c) L_3(x, y, z) = (x + 1, y - 1, z - 2).$

3. (6 points) What are the eigenvalues for the matrix A below? Find a basis for each eigenspace associated to the *real eigenvalues only*. Is A diagonalizable or not? Is A invertible? Find a basis for the row space of the matrix A.

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

4. (3 points) Decide whether the following statements are true or false (give a short justification if you want full points!):

   (a) The set of all $n \times n$ symmetric matrices is a subspace of all $n \times n$ matrices with the usual sum and scalar product.

   (b) If A is an $n \times n$ non-singular matrix then if t is an eigenvalue of A then $1/t$ is an eigenvalue of A^-1.

   (c) The system $Ax = b$ has a solution if and only if b is in the column space of A.

   (d) If A is a singular $n \times n$ matrix, then A^3 is singular.

   (e) Every linear system $Ax = 0$ where A is an $m \times n$ matrix has a non-trivial solution (different from $x = 0$) if $m < n$.

   (f) For any $m \times n$ matrix, with $m < n$, the dimension of the column space is bigger than the dimension of the row space.