Math 167 (De Loera) **Homework 1**
Due date: October 15, 2014

INSTRUCTIONS

This homework has many problems. By presenting solutions of ALL problems you will receive 1point. 3 problems will be marked for correctness for the remaining 4 points. MATLAB exercises need to be submitted via SMARTSITE using the assignment boxes.

Write legibly preferably using word processing if your hand-writing is unclear. Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. Read Chapter 1 & 2 of Eldén.

2. Given the prerequisites for this class (MAT 22A or 67) and the material outlined in the course syllabus, the following concepts are important for you to understand. We will discuss these concepts in my lectures, but do not wait for my lectures. If your memory is vague on these subjects, review them yourself now!

    - Solving linear systems
    - LU decomposition
    - Equivalences of non-singular matrices
    - Rank and span
    - Kernel, domain, range
    - Linear combinations
    - Basis vectors, change of basis
    - Column space and row space
    - Orthogonal vectors, Gram-Schmidt orthogonalization
    - Dot product and outer product
    - Eigenvalues, eigenvectors, diagonalization

3. Consider the following matrix:

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 7 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

    (a) View this matrix as a linear transformation between two vector spaces. Identify the domain and the range space (codomain) of this linear transformation.

(b) Identify the image of this linear transformation.

(c) Write a basis for the image of this linear transformation.

(d) Identify the kernel of this linear transformation.

(e) Write a basis for the kernel of this linear transformation.

(f) What is the column space of this linear transformation?

(g) What is the row space of this linear transformation?

(h) What is the rank of the matrix A?

4. Solve problems 1.4, 1.19, and 1.20 in Chapter 1 of Moler's MATLAB book.

5. Solve problems 2.2, 2.4, 2.7, 2.9 in chapter 2 of Moler's MATLAB book.

6. Recall that given an (undirected) graph G with n vertices $v_1, \ldots, v_n$, we define the **adjacency matrix** of G with respect to the labeling $v_1, \ldots, v_n$ of the vertices as being the $n \times n$ matrix $A_G = (a_{ij})$ whose entry $a_{ij} = 1$ if there is an edge between vertex v_i and v_j and zero otherwise.

   Write a formal proof of the following theorem we discussed in class: **Theorem:** If A_G is the adjacency matrix of a graph G (with vertices $v_1, \ldots v_n$), the (i, j)-entry of $(A_G)^r$ represents the number of distinct walks from vertex v_i to vertex v_j in the graph of length r.

7. **True or False:**

   (a) Let P be a permutation matrix their determinant is always 1.

   (b) If the matrix A has eigenvalues $2, 2, 5$ then the matrix is invertible.

   (c) If Q is an orthogonal matrix then Q^{-1} is an orthogonal matrix

   (d) If A has eigenvalues $1, 1, 2$ then A is diagonalizable

   (e) If S is a matrix whose columns are linear independent eigenvectors of A then A is invertible

   (f) If A is PSD and Q is orthogonal $Q^T AQ$ is PSD

   (g) If A is PSD and Q is orthogonal $Q^T AQ$ is diagonal