

INSTRUCTIONS

This homework has many problems. By presenting solutions of ALL problems you will receive 1 point. 2 or 3 problems will be marked for correctness for the remaining 4 points. MATLAB exercises need to be submitted via SMARTSITE using the assignment boxes.

Write legibly preferably using word processing if your hand-writing is unclear. Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. Read Chapter 4 of Eldén, Read Chapter 5 of Moler's online book.
2. Given b a vector in R^2 , is the closest point on a line L unique when the distance is measured in (a) The Euclidean norm? (b) the 1-norm? (c) the ∞ -norm?
3. In the least squares method the matrix $P = A(A^T A)^{-1}A^T$ is an important matrix. Prove that $P^2 = P$ and that it is symmetric. Prove that the range of P and A coincide.
4. Is the least squares solution always unique? HINT: What if A has linear dependent columns?
5. Find the vector v inside the subspace generated by the vectors $(0, 0, 1, 1), (2, 1, 1, 1)$ that minimizes the $\|v - (0, 3, 1, 2)\|$.
6. The proprietor of an internet travel company compiled data relating annual profit of the firm to annual advertisement expenditure (both in thousands of dollars):

annual advertising expenditure	12	14	17	21	26	30
Annual Profit	60	70	90	100	100	120

- (a) determine the equation of the least squares line (b) Plot the data and the least square line. Estimate the profit when the annual advertising budget is 50,000. (c) What about 100,000?

x	1	1	2	2	3	3
y	1	2	1	2	2	4
z	3	6	11	-2	0	3

7. a) Determine the best plane which fits the data above in the least-squares sense b) How would you answer the data if the plane is constrained to go through the point $x = 2, y = 2, z = 0$?
8. Find the function of the form $ae^t + be^2t$ that best approximates t^2 in the least-square sense based on the sample points 0, 1, 2, 3, 4. (a) What is the least-squares error? (b) how much better can you do if you include a constant term, i.e., $ae^t + be^2t + c$?

9. True or False: a) If A, B are orthogonal matrices then $A + B$ is orthogonal too. b) A orthogonal implies $\|Ax\| = \|x\|$. c) A orthogonal if and only if $\|Ax - Ay\| = \|x - y\|$. d) Let A orthogonal matrix and x_1, x_2, \dots, x_n be an orthonormal basis for R^n , then Ax_1, \dots, Ax_n is an orthonormal basis for R^n too.
10. Use the Gram-Schmidt procedure to find an orthonormal basis for the row space, column space and the nullspace of the matrix A below

$$\begin{pmatrix} 2 & -2 & -5 & -3 & 1 & 2 \\ 2 & -1 & -3 & 2 & 3 & 2 \\ 4 & -1 & -4 & 10 & 11 & 4 \\ 0 & 1 & 2 & 5 & 4 & 0 \end{pmatrix}$$

11. Solve problem 5.8 in Chapter 5 of Moler's MATLAB book.
12. Solve problem 5.11 (a,b) in Chapter 5 of Moler's MATLAB book.