Math 167 (De Loera) **Name:**
Mid-term exam 1 **Student ID#**
February 6, 2012

INSTRUCTIONS

(1) READ INSTRUCTIONS CAREFULLY!!!
(2) DO NOT TURN THIS PAGE UNTIL INSTRUCTED TO DO SO
(3) FILL IN THE INFORMATION ON THIS PAGE (your name, etc) NOW!!
(4) SHOW YOUR WORK on every problem. Correct answers with no support work will not receive full credit.
(5) PLEASE WRITE LEGIBLY. Be organized and use the notation appropiately.
(6) NO EXTRA ASSISTANCE ALLOWED. Assistance from classmates, notes, books is prohibited.
(7) YOU MAY USE THE BACK SIDE OF THE PAPER TOO.

1. (6 points) Decide whether the following statements are true or false, giving a brief justification (all problems taken from odd problems in sections 1.6-2.4):
   (a) A square matrix with 1s down the main diagonal is invertible.
   (b) If A is square matrix, then if $A^2 + A = I$ then $A^{-1} = A + I$.
   (c) If A square matrix with all diagonal entries zero, then A is singular
   (d) Suppose A is an arbitrary matrix, whose columns are linearly independent, then $Ax = b$ has exactly one solution for all b.
   (e) If v_1, v_2, v_3, v_4 is a basis for the vector space R^4, and if W is a subspace, then some subset of the v's is a basis for W.
   (f) If the row space equals the column space then $A^T = A$.
2. (10 points) Prove that the matrix norm based on the 1-norm is given by $||A||_1 = \max_{1 \leq j \leq n} ||a_j||_1$ where a_j denotes the j-th column of A. Is the bound attained at a particular vector? Show one if so!
3. (4 points) (2.2. (51)) The nullspace of a 3×4 matrix is the line through $(2, 3, 1, 0)$.
   (a) What is the rank of A and the complete solution of $Ax = 0$?
   (b) What is the exact row reduced echelon form R of A?

4. (10 points) The $n \times n$ "checkerboard matrix" C_n has $a_{ij} = 0$ when $i + j$ is even and $a_{ij} = 1$ otherwise.

(a) What is the picture of C_8?

(b) Find a basis for $Nullspace(C_8)$?

(c) What is the rank of C_n as a function of n? What is the dimension of its left nullspace?

(d) Are all the $n \times n$ permutation matrices linearly independent (as vectors of the vector space $R^{n \times n}$)?

(e) Write C_4 as a linear combination of 4×4 permutation matrices. Can you do the same for C_8 (i.e., using 8×8 permutation matrices)?