Math 22A (De Loera) **Homework 1**
Due date: Monday, Jan 14 2019

INSTRUCTIONS

This homework is worth 5 points. It consists of the WEBWORK problems I assigned PLUS the problems I wrote below. By presenting solutions of ALL WEBWORK problems you will receive up to 4 points (depends of how many you get right). One problem below will be graded for correctness for 1 point. You will get one more point if you do all the problems below.

At the bottom of this homework I also included challenge 1. You need to be submit this via CANVAS by Jan 14th).

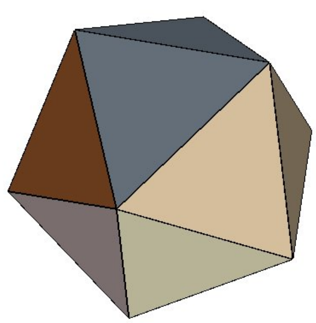
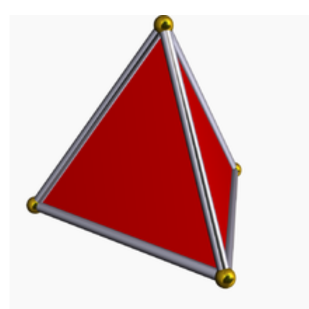
Write legibly but preferably use word processing if your hand-writing is unclear. Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. AS ALWAYS: Do all WEBWORK problems assigned in Strang W00 and Strang W01. Read Chapter 1 of Strang. (worth 4 points).

2. Draw 3 vectors in R^3, u, v, w so that their linear combinations fill only a line. Similarly, find other three vectors that fill only a plane, and a final set of three vectors so that their linear combinations are the entire space.

3. If someone tells you that the vectors v has norm 5 and the vector u has norm 3. What are the smallest and largest value of $||u - v||$? What are the smallest and largest values of $v \cdot u$?

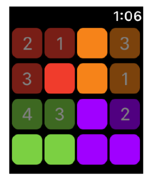
4. Here is a system of equations coming from geometry: A *trianguled sphere* is composed of v vertices, e edges and f tri- angular faces (see picture for example) glued to form a ball shape (see two examples below in picture). World-famous mathematician Leonard Euler (in picture) showed that $v - e + f = 2$ always (as long as the polyhedron has no holes! It is ball shaped, not doughnut shape).

The shape is composed of triangles, line segments and points glued together. Every edge is in exactly two triangles, every triangle has three edges. Thus $3f = 2e$. Your cousin gives you a polyhedron with $f = 288$ (288 triangles!). Write down a system of linear equations and figure out how many vertices and edge does it have.

5. A new trendy game in a popular watch is the following:

The rules are that in each of the four 2×2 sub-squares, in each of the four rows and each of the four columns, the entries 1 to 4 have to appear and so add up to 10. The game initially gives 4 numbers. We have already started to enter 4 more numbers. There are still 8 missing. Introduce 8 variables and write down a system of 12 linear equations for these 8 variables, then solve the system.



Challenge 1: Linear Algebra for Networks Analysis: (1 point)

Given an (undirected) graph or network G with n vertices $v_1, \ldots, v_n$, define the **adjacency matrix** of G with respect to the labeling $v_1, \ldots, v_n$ of the vertices as being the $n \times n$ matrix $A_G = (a_{ij})$ whose entry $a_{ij} = 1$ if there is an edge between vertex v_i and v_j and zero otherwise. This matrix encodes all the information of the network (e.g., the internet!).

Challenge: Explain what is the meaning of the (i, j)-entry of $(A_G)^r$ (A_G multiplied with itself r times). Justify your answer in order to be very convincing. Answers without careful justification receive zero points!