Math 22A (De Loera) **Homework 3 + Challenge 2**
Due date: Wednesday, February 6 2019 at 11:59pm

INSTRUCTIONS

This homework is worth 5 points. It consists of the WEBWORK problems I assigned PLUS the problems I wrote below.

Write legibly but preferably use word processing if your hand-writing is unclear. If you send a photograph it better be legible and send one photo per problem with name of problem clearly indicated. Be organized and use the notation appropriately. Show your work on every problem. Correct answers with no support work will not receive full credit.

1. AS ALWAYS: Do all WEBWORK problems assigned. This time they are in Strang W03 and W04. Finish reading Chapter 2 of Strang.

2. Use the techniques you learned to solve the following problems in Section 2.4 of textbook: (15, 16, 32, 34). Justify your answer or get zero points!

3. Use the techniques you learned to solve the following problems in Section 2.5 of textbook: (1, 9, 11, 29, 31).

4. Use the techniques you learned to solve the following problems in Section 2.6 of the textbook: (4,8,24).

5. Matrices are very powerful because they have several meanings: We saw they are representations of systems of linear equations (e.g., used in elimination), we also saw matrix multiplication is the same as operations on matrices (row addition is multiplication). But here is one more powerful way to think of matrices: Given an $m \times n$ matrix A, we can associate a map from R^n into R^m which takes a vector x in R^n and maps it to the vector Ax (a vector in R^m). Such maps are call *linear transformations* or *linear maps*. In this exercise you will play with matrix multiplication as maps, or transformations of vectors!

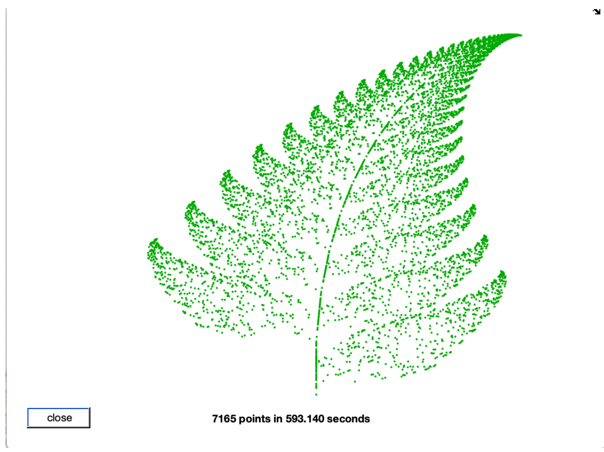
   (a) Describe the linear transformations $x \to Ax$ given by each of the following matrices
   $A_1 = \begin{pmatrix} 4 & 1 \\ 8 & 2 \end{pmatrix}$, $A_2 = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$, $A_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $A_4 = \begin{pmatrix} 0 & 1 \\ 0 & 4 \end{pmatrix}$, $A_5 = \begin{pmatrix} 3 & -2 \\ -6 & 4 \end{pmatrix}$.
   To help on your explanations, make five pictures of what happens to the triangle with corners $(0,0), (1,0)$ and $(0,1)$.

   Explain what happens to the picture if the transformation is $x \to Ax + b$, where $b = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$.

   (b) What 2×2 linear transformations of the vector $\begin{pmatrix} x \\ y \end{pmatrix}$ produce i) $\begin{pmatrix} x \\ y \end{pmatrix}$? ii) $\begin{pmatrix} 0 \\ y \end{pmatrix}$? iii) $\begin{pmatrix} x \\ y+x \end{pmatrix}$?
   iv) $\begin{pmatrix} 7x \\ \frac{1}{3}y \end{pmatrix}$?

CHALLENGE 2: (IMPORTANT: This is due on February 8th 11:59pm!) This is a MATLAB challenge to make you think more deeply about the important connections between *computer graphics* and *linear algebra*.

The figure you see below of a fern (type of plant) was generated using MATLAB. It comes from repeated linear transformations of a point in the plane using 4 different matrices, plus translations. The transformations are applied at random.



Your challenge is to

1. Read, experiment, and understand the MATLAB code I sent you (fern.m and fernfinite.m). The first one runs with the command fern and runs until you stop it (the picture above was about 9 mins). finitefern(n) generates n points and plots a figure too. The command finitefern(n,'s') shows the generation of points one at a time.

2. Modify the code to flip the fern by interchanging its x and y coordinates. What happens to the fern if you change the only non-zero entry in the matrix $A4$? You must submit your code and a picture.

3. Modify the code. Start with $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$. Always apply $A = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix}$ (instead of the 4 choices in fern.m) but the translation vector b is chose at random with equal probability from among the three vectors
$\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1/2 \\ 0 \end{pmatrix}$ $\begin{pmatrix} 1/2 \\ \sqrt{3}/4 \end{pmatrix}$. You must submit your code and a picture.