1. A ring is called Noetherian (in honor of Emmy Noether!) when every ideal is finitely generated (e.g. We are proving $K[x_1, \ldots, x_n]$ is such a ring. Show that a ring is Noetherian if and only if it contains no infinite ascending chain of ideals (i.e. ideals $I_1 \subset I_2 \subset \ldots$ where $I_n$ is contained properly in $I_{n+1}$).

2. Show that grevlex is a monomial order according to the definition.

3. Let $>$ be a monomial order on the polynomial ring $S = K[x_1, x_2, \ldots, x_n]$. Let $f, g \in S$, is it true that $\text{LT}(fg) = \text{LT}(f)\text{LT}(g)$?

4. Is $G = \{x^4y^2 - z^5, x^3y^3 - 1, x^2y^4 - 2z\}$ a Gröbner basis for the idea $< G >$ with respect to grlex order? Let $I = < g_1, g_2, g_3 >$ where $g_1 = xy^2 - xz + y$, $g_2 = xy - z^2$, and $g_3 = x - yz^4$. Using the lex order, give an example of $g \in I$ such that $\text{LT}(g)$ is not in the monomial ideal $< \text{LT}(g_1), \text{LT}(g_2), \text{LT}(g_3) >$. 
