Algebra, Math 250B
Homework four, Due Feb 24, 2017

1. Let K be a field, what is $Spec(K[x,y])$?

2. Prove the lemma we stated in class that the ring homomorphisms under the Zariski topology become continuous maps.

3. Consider the homomorphism of polynomial rings

   $\phi : \mathbb{Q}[x,y,z] \to \mathbb{Q}[t]$

   given by $x \to t^2$, $y \to t^3$, $z \to t^5$ and where $\phi(q) = q$ for all rational numbers. Let I be the kernel of this map. Prove of disprove:

   (a) I is a prime ideal.

   (b) I is a maximal ideal.

   (c) I is a principal ideal.

   Can you find who I is explicitly?

4. From Rotman page 301-302: Problems 5.5, 5.10, 5.11

5. From Rotman Page 311-312: Problems 5.17, 5.23

6. From Rotman Page 315-316: Problems 5.28, 5.30, 5.32

7. True or false? Why?

   (a) Every subring of $\mathbb{Q}[x_1, x_2]$ is finitely generated.

   (b) Every subring of $\mathbb{Q}[x_1, x_2]$ is a unique factorization domain.