Algebra, Math 250B
Homework one, Due January 18

1. From Hungerford page 120-121: Problems 3,9,11,18.
2. From Hungerford page 133-135: Problems 2,17,20,23.
3. From Hungerford page 140-141: Problems 3,5,8,9.
4. For an ideal I in the polynomial ring $Q[x_1, \ldots, x_n]$ define the *variety* of I to be

$$V(I) := \{a \in \mathbb{C}^n : f(a) = 0 \ \forall \ f \in I\}.$$

   Prove or disprove:

   (a) $V(I + J) = V(I) \cap V(J)$.
   (b) $V(I \cap J) = V(I) \cup V(J)$.
   (c) For any finite subset A of $\mathbb{C}^n$ there exist an ideal I with $V(I) = A$.

5. Prove the following two statements:

   (a) $x^4 + 1$ is irreducible in $\mathbb{Q}[x]$.
   (b) $x^4 + 1$ is reducible in $\mathbb{Z}_p[x]$ for any prime number p.