Algebra, Math 250B
Homework two, Due Jan 27

1. From Hungerford page 240-242: Problems 2,8,13,14,16,24,25.

2. From Rotman page 170-171: 2.93, 2.99.

3. If $f(x) = a_n x^n + \ldots + a_0$ has (all non-zero) roots $\alpha_1, \ldots, \alpha_n$. Note that the coefficients of f are elementary symmetric functions of the roots. Using this fact find a polynomial that has roots $c\alpha_1, \ldots, c\alpha_n$, and another polynomial that has roots $\frac{1}{\alpha_1}, \ldots, \frac{1}{\alpha_n}$,

4. Let K be the splitting field of $x^{12} - 1$ over $\mathbb{Q}$. Calculate the degree of the extension $K/\mathbb{Q}$ and find a vector space basis.

5. Compute the minimum polynomial for $\sqrt{2}\sqrt[3]{5}$ using the Sylvester resultant techniques. Can you explain how to generalize this? Say a, b are integers such that $\sqrt[3]{a}$ and $\sqrt{b}$ are not rational numbers. Using the discussion in class, compute the minimal polynomial of $\sqrt[3]{a} + \sqrt{b}$ over the rational numbers in terms of a, b.