Or better:
"the groupoid of $F$-structured finite sets labelled by objects of $Z_0"."

Yes! Let's see how this works. Recall the weak quotient
\[
\frac{F_n \times Z_0^n}{n!}
\]
has the same objects as $F_n \times Z_0^n$, so its objects of this is:

- an ordered pair consisting of
  - an $F$-structure on the $n$-elt set
  - an $n$-tuple of objects in $Z_0$

i.e.

- an $F$-structure on $n$ & a way of labelling its elements by objects of $Z_0$.

**Example:**

$F =$ being a totally ordered set

\[
\begin{array}{c}
1_R^G \xrightarrow{f} G_1^B \\
Z_0 = \ x \ \ y \ \ z \ \ w \\
B_1^G \ \ g \ \ g^2 = 1, g_1 = 1 \\
\end{array}
\]

$n = \{0, 1, 2, 3\} = 4$

An object of $\frac{F_4 \times Z_0^4}{4!}$: 3 1 0 2

R B R G
The morphisms of \( \frac{F_n \times Z_0^n}{n!} \) are generated by morphisms in \( F_n \times Z_0^n \) & morphisms coming from elts of the permutation group \( n! \), & satisfying certain relations we called (1) & (2) last time. In our example, a typical morphism in \( F_n \times Z_0^n \) is

\[
\begin{array}{cccc}
3 & 1 & 0 & 2 \\
\text{R} & \text{B} & \text{R} & \text{G} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\text{f} & \text{g} & \text{h} & \text{f'} \\
\text{G} & \text{B} & \text{R} & \text{R} \\
3 & 1 & 0 & 2
\end{array}
\]

while a typical morphism coming from \( 4! \) is:

\[
\begin{array}{cccc}
6 & 0 & 3 & 2 \\
\text{B} & \text{R} & \text{G} & \text{R} \\
\downarrow & \downarrow & \downarrow & \downarrow \\
\text{f} & \text{g} & \text{f'} \\
1 & 0 & 3 & 2
\end{array}
\]

-a permutation in \( 4! \)

The relations (1) & (2) are obvious in this picture, e.g.: