Davis Math Lab Project Report, Spring 2026

CEMP for SE(d) Synchronization

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1. Introduction

Rigid-motion synchronization, also known as SE(d) synchronization, is an important problem in computer vision, robotics, and image analysis. The objective is to recover absolute positions and orientations from noisy relative measurements between objects or sensors. Applications include image alignment, structure-from-motion, simultaneous localization and mapping (SLAM), sensor network localization, and cryo-electron microscopy. Existing synchronization methods often suffer when measurements contain large amounts of corruption or outliers. The Cycle-Edge Message Passing (CEMP) algorithm has previously demonstrated strong robustness for synchronization problems on compact groups such as SO(d). However, extending CEMP to the Special Euclidean group SE(d) is challenging because SE(d) is non-compact and lacks a natural bi-invariant distance metric. The goal of this project is to extend the CEMP framework to SE(d) synchronization. Specifically, we develop a modified cycle-consistency metric and message-passing procedure that jointly utilize rotation and translation information. We also investigate how the resulting corruption estimates can be combined with spectral synchronization methods to improve robustness under noisy and corrupted measurements.

2. Progress

During Spring 2026, our work focused on extending the Cycle-Edge Message Passing (CEMP) framework from compact groups such as SO(d) to the Special Euclidean group SE(d). This required both theoretical development and computational experimentation. The first stage of the project involved understanding the mathematical structure of rigid-motion synchronization. In synchronization problems, each object is associated with an unknown absolute rigid motion consisting of a rotation and a translation. Only noisy pairwise relative motions are available, and the goal is to recover the absolute motions. We studied the existing CEMP framework and identified the main challenge preventing its direct application to SE(d): the group is non-compact and does not possess a natural bi-invariant metric. To overcome this limitation, we introduced a modified distance metric that combines rotational and translational discrepancies. The metric was designed to be left-invariant, allowing it to preserve key properties required by the CEMP algorithm. We carefully analyzed the mathematical properties of this metric and verified that it provides a meaningful measure of corruption for rigid-motion measurements. After defining the new metric, we modified the cycle consistency formulation used in the original CEMP algorithm. We derived a new cycle-edge inconsistency measure and showed that, under clean-cycle assumptions, the modified quantity remains an exact estimator of edge corruption. We further studied the theoretical guarantees of the method and demonstrated that the convergence behavior of CEMP can be preserved under reasonable assumptions. The second major component of the project involved algorithm implementation. We implemented the modified CEMP algorithm in Python and tested its performance on simulated synchronization problems. The implementation included cycle generation, message passing, corruption estimation, and iterative weight updates. We also implemented visualization tools that allowed us to compare estimated corruption levels against ground-truth corruption values. To evaluate the effectiveness of the proposed method, we generated synthetic SE(2) datasets with varying noise levels and corruption probabilities. Numerical experiments showed that the estimated corruption levels closely matched the true corruption levels when noise was moderate. Scatter plots comparing estimated and true corruption values demonstrated strong agreement across a wide range of experimental settings. We further analyzed the distributions of estimated corruption levels under different corruption and noise regimes. The results revealed that the distribution of corruption estimates contains useful information about data quality. Finally, we combined the proposed CEMP framework with the spectral synchronization algorithm EIG. Corruption estimates generated by CEMP were converted into edge weights and incorporated into the synchronization process. Experimental comparisons demonstrated that the combined CEMP-EIG method significantly outperformed traditional EIG-IRLS approaches under high corruption conditions. Overall, the project successfully established a theoretical framework, developed practical algorithms, and validated the effectiveness of the proposed approach through extensive numerical experiments.

3. Future Plans

Several directions remain for future development. First, we plan to explore more sophisticated weighting strategies that can further improve the interaction between CEMP corruption estimates and synchronization algorithms. Second, although our current experiments focus primarily on SE(2), future work will extend the framework to higher-dimensional groups such as SE(3), which are commonly encountered in computer vision and robotics applications. Third, we intend to evaluate the method on real-world datasets, including image alignment and structure-from-motion tasks. Such experiments will help assess practical performance beyond simulated environments. Finally, we aim to improve computational efficiency and scalability so that the algorithm can be applied to larger synchronization problems encountered in practical applications.

4. References

Yang, J., & Shi, Y. CEMP for SE(d) Synchronization, 2026.

Arrigoni, F., et al. Spectral Motion Synchronization in SE(3), 2021.

Lerman, G., & Shi, Y. Robust Group Synchronization via Cycle-Edge Message Passing, 2022.