

# Curves

For the  
mathematically  
curious

Graduate Mentor:  
Isaiah Williams

*“What is a curve? Everyone knows what a curve is, until they have studied enough mathematics to become confused through the countless number of possible exceptions.”*

— Felix Klein (1958)

## Seminar Description

Any curve that has a name has a story. In this seminar, students will explore ten remarkable curves – their shapes, equations, and surprising properties – alongside the historical moments and personalities that brought them to life. We will examine both the mathematical structure of these curves, from curvature to fractal dimension, and the ideas they helped shape, from classical geometry to modern cryptography.

## Prerequisites

Calculus I and II. Beyond that, curiosity, since it will help you choose a curve to present on!

## Format

This is a student-led seminar. At the start of the seminar, one or a pair of students will give a short presentation ( $\leq 15$  minutes) about a curve of their choosing. The format of each presentation is flexible, but should follow three guiding questions:

- (1) What historically inspired the study of this curve?
- (2) What mathematical ideas are needed to understand this curve?
- (3) What are the defining properties of this curve?

Afterward, we will break out into small groups and collaborate on a set of guided exercises that I will design to reinforce the key ideas and properties of the curve. I will be available throughout the week to discuss ideas, clarify questions, and help presenters refine their material.

## Schedule

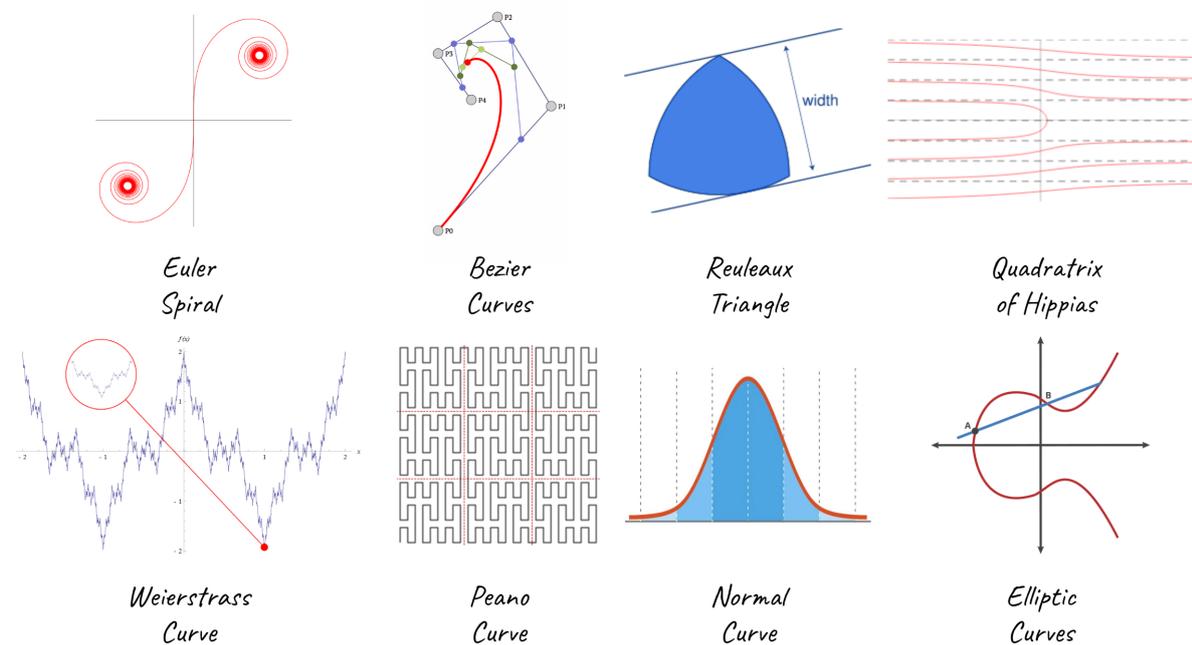
We will meet once a week for one hour (on Tuesday or Thursday afternoons, depending on availability). Our first meeting will include introductions and I will give an example presentation

to give you a concrete idea of the expected format and level of depth. After that, the seminar will revolve around student presentations, loosely progressing from classical geometric curves toward more analytically and algebraically sophisticated examples.

## Material

Julian Havil provides examples of many interesting curves in *Curves for the Mathematically Curious: An Anthology of the Unpredictable, Historical, Beautiful, and Romantic*, including

- **Euler Spiral** — A curve whose curvature increases linearly with arc length [1]
- **Bézier Curves** — Polynomial parametric curves determined by a finite set of points [1, 2, 3]
- **Reuleaux Triangle** — A non-circular curve of constant width [1, 2]
- **Quadratrix of Hippias** — A curve created to solve the classical geometric problems of angle trisection and squaring the circle [1, 2]
- **Weierstrass Curve** — A curve that is continuous everywhere but differentiable nowhere [1]
- **Peano Curve** — A continuous curve that fills a unit square [1, 2, 3, 4, 5]
- **Normal Curve** — The graph of the Gaussian density function, central to probability [1]
- **Elliptic Curves** — Smooth cubic curves endowed with a natural group structure, foundational in number theory and modern cryptography [1, 2, 3]



There are **many other curves beyond those listed in the book**, and you are encouraged to find one that genuinely interests you. The only requirement is that the curve be planar (i.e. defined in two-dimensional space), since familiarity with multivariable calculus is not assumed.

## Contact

If you have any questions or comments, feel free to reach out to me at [ipwilliams@ucdavis.edu](mailto:ipwilliams@ucdavis.edu).