

Markov Chains

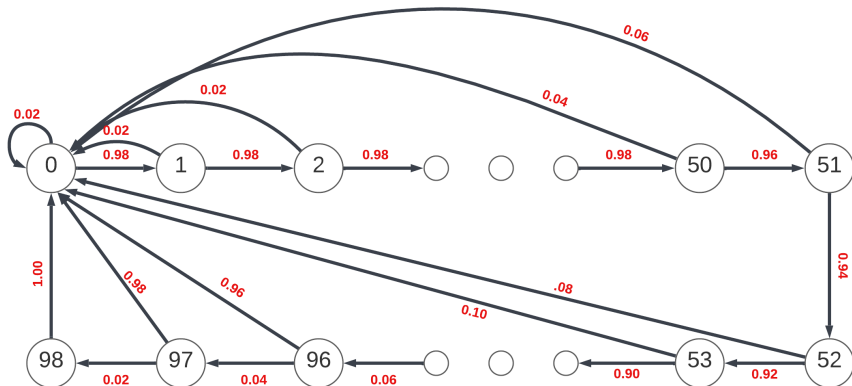
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If you have not obtained a 6-star after 50 rolls, the odds of getting a 6-star increase from 2% to 4%. If after 51 rolls you have not obtained a 6-star, the odds increase again from 4% to 6%. This process repeats till the probability reaches 100%.

Graphical Representation of Arknights

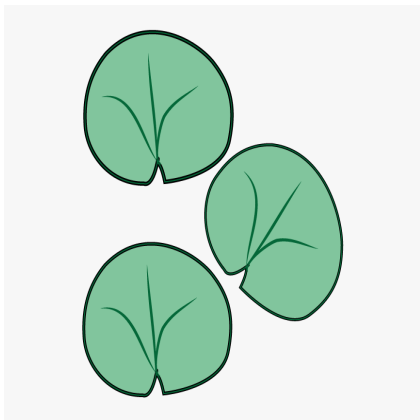
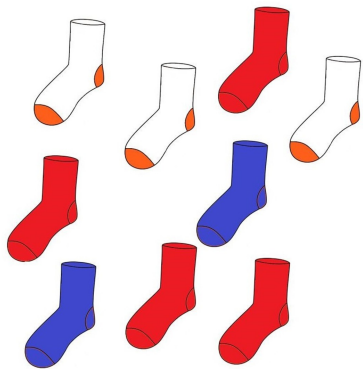


What is a Markov Chain?

Definition

A sequence of random variables is Markov Chain with state space χ and transition matrix P if the probability of getting from one state to another is dependent only on the current state.

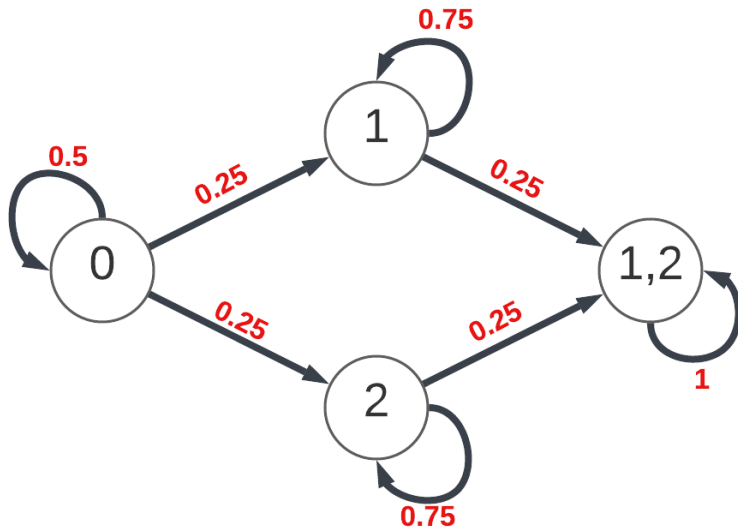
Example and Counterexample



Coupon Problem

You are collecting coupons. For simplicity there are two coupons that you want to collect and four coupons in total. Each day a coupon will appear with each one having an equal probability.

Graph of Coupon Problem



Solution to Coupon Problem

$$\begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \\ 0 \end{bmatrix}$$

Solution to Coupon Problem

$$\begin{bmatrix} 0.5 & 0 \\ 0.25 & 0.75 \\ 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$$

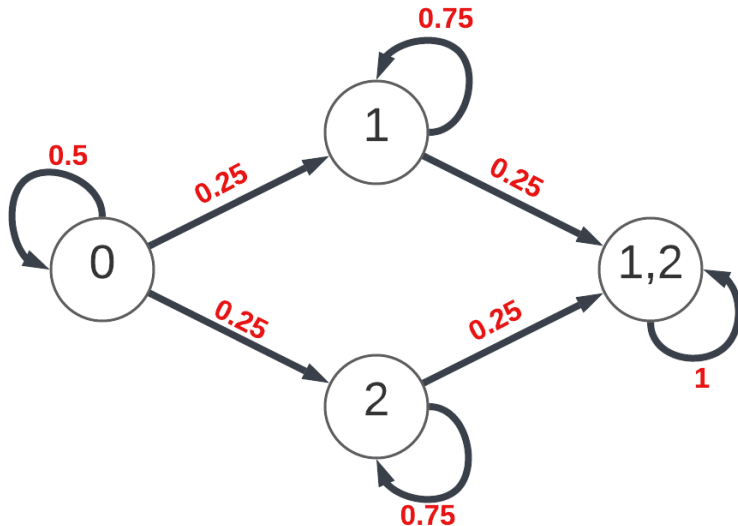
Solution to Coupon Problem

$$\begin{bmatrix} 0.5 & 0 & 0 \\ 0.25 & 0.75 & 0 \\ 0.25 & 0 & 0.75 \\ 0 & 0.25 & 0.25 \end{bmatrix}$$

Solution to Coupon Problem

$$\begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0.25 & 0 & 0.75 & 0 \\ 0 & 0.25 & 0.25 & 1 \end{bmatrix}$$

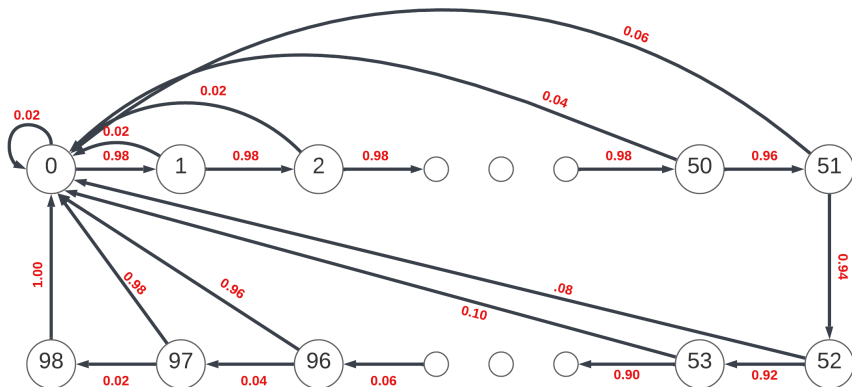
Solution to Coupon Problem



Solution to Coupon Problem

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0.5 & 0 & 0 & 0 \\ 0.25 & 0.75 & 0 & 0 \\ 0.25 & 0 & 0.75 & 0 \\ 0 & 0.25 & 0.25 & 1 \end{bmatrix}^n$$

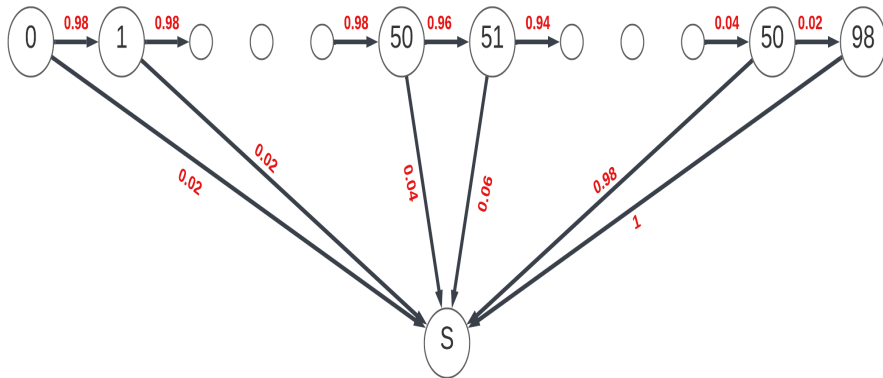
Graphical Representation of Arknights



Simplifying the Problem

How many pulls between 6-stars on average?

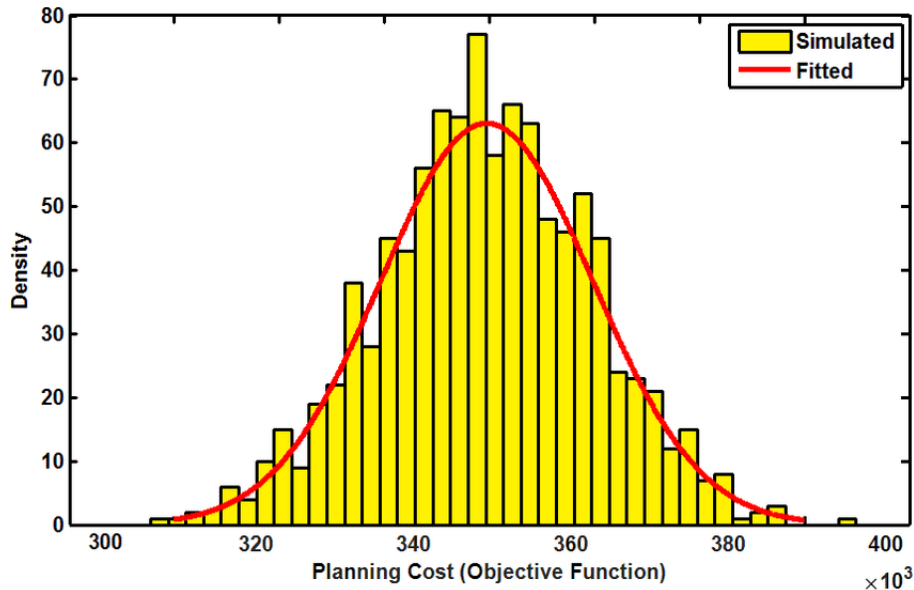
Graph of Simplification



Expected Hitting Time

$$\sum_{\chi} (\text{Probability of getting to that state}) * (\text{Probability of getting to the goal state}) * (\text{Number of steps to get there})$$

Monte-Carlo Simulations



Monte-Carlo Results

35	0.69989			466	13.17284
36	0.7183			467	13.19719
37	0.73822	211	5.7996	468	13.23773
38	0.76044	212	5.82708	469	13.25391
39	0.78062	213	5.85051	470	13.29094
40	0.80341	214	5.88814	471	13.31137
41	0.82907	215	5.91264	472	13.33981
42	0.84422	216	5.94003	473	13.38317
43	0.86201	217	5.96932	474	13.402
44	0.87882	218	5.9963	475	13.44508
45	0.90028	219	6.02929	476	13.44992
46	0.91938	220	6.05792	477	13.48039
47	0.94199	221	6.07576	478	13.52044
48	0.96418	222	6.109	479	13.54604
49	0.9767	223	6.13948	480	13.57886
50	1.00015	224	6.16879	481	13.59425
51	1.02798	225	6.20857	482	13.62688
52	1.05793	226	6.22304	483	13.65163
53	1.10098	227	6.26296	484	13.69041
54	1.14699	228	6.28901	485	13.73026
55	1.19176	229	6.32284	486	13.74239
56	1.2485	230	6.33798	487	13.77805
57	1.29964	231	6.37496	488	13.79682
		232	6.40593		

Data Comparison

Pull Count	Expected Result	Actual Result
139	3.726	4
81	2.067	2
103	2.648	2
76	1.914	1
115	3.011	2
75	1.914	2
71	1.904	2
103	2.648	2
190	5.195	6
20	0.400	1
40	0.977	1
160	4.323	5
91	2.336	3

Data Comparison

Pull Count	Expected Result	Actual Result
138	3.695	5
70	1.778	3
126	3.360	3
140	3.726	3
180	4.905	4