

Finite Groups: Coprime Extensions and Hall Subgroups

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Seminar Description: While groups are covered in detail in the MAT 150 series, there are many theorems and techniques essential to the study of finite groups that students are not shown. In this seminar, we aim to learn such topics. In particular, our main goal will be to cover the Schur-Zassenhaus Theorem, which trivializes many group extension problems, and Hall π -subgroups, which are a generalization of Sylow p -subgroups, but for multiple primes. This course will also be a presentation course, giving students the chance to develop their mathematical presentation skills in a consequence-free environment.

Learning Objectives:

- Students will be able to create, plan, and run an expository math presentation.
- Students will be able to read graduate level textbooks, internalize the material, and share this material with their peers.
- Students will be able to answer questions asked during their presentation, or if they do not know the answer to a question, gracefully handle it and continue with the talk.
- Students will learn Sylow's theorems, solvable groups, semidirect products, Schur-Zassenhaus theorem, and Hall subgroups.

Prerequisites: MAT 150A or equivalent knowledge of group theory. Experience with Sylow's theorems is encouraged but not required.

Sources: I. Martin Isaacs' *Finite Group Theory*, Chapters 1A-1C, 3A-3D

Format: In any academic (or even industry) career, giving presentations is an oft-overlooked yet important skill. It is good to develop this skill early! To that end, this is a presentation-style course. This means that at all meetings except the first, *students will be expected to present* topics from the textbook, selected by the instructor. If no students are available to present during a given week, class will not be held that week.

If you are presenting:

- Please prepare a 50 minute long presentation to fill class time, or collaborate with other students to fill the 50 minutes
- Generally the best options are either a Beamer (like powerpoint for latex) or a blackboard presentation. (I prefer the latter, but do what you prefer!)
- If you need help compiling a presentation or if you are struggling with the material, you are welcome (and encouraged) to reach out to me, and I will do my best to help.
- While I prefer you to ask questions about the material before presenting, if there is something you don't know, you can always ask me during your presentation, and I'll try my best to answer.

- Likewise, you are expected to answer questions from your peers. If you truly don't know the answer to a question, that's OK, and you can always pivot to me.

If you are not presenting:

- You are expected to be respectful to the presenters while they are presenting.
- You are encouraged but not required to read the material in the book before coming.
- You are strongly encouraged to take notes in class, so that when it is your turn to present you are not lost.
- If you are having trouble understanding something, ask questions! You get out what you put in to this course, and asking questions is the best way to learn.

Outline: The outline is subject to change based on the length of student presentations and the difficulty of the material. The schedule is subject to change depending on the difficulty of the material and the pace at which students present. If we don't hit all the topics, that's okay! What follows is a rough outline of what I would ideally like to cover.

- Week 1-2: Group theory and Sylow theory review (Isaacs 1A-1C)
- Week 3: Solvable Groups (Isaacs 3B)
- Week 4-5: Semidirect Products (Isaacs 3A)
- Week 6-7: Schur-Zassenhaus (Isaacs 3B)
- Week 8-9: Hall Subgroups (Isaacs 3C-3D)