Math 115A Homework 4

1) Find all positive integers \( m \) for which the following statements are true.
   a) \( 13 \equiv 5 \pmod{m} \)
   b) \( 10 \equiv 9 \pmod{m} \)

2) Let \( a, b \in \mathbb{Z} \) such that \( a \equiv b \pmod{m} \). If \( n \) is a positive integer such that \( n|m \), prove that \( a \equiv b \pmod{n} \).

3) Let \( a, b \in \mathbb{Z} \) such that \( a \equiv b \pmod{m} \). If \( c \) is a positive integer, prove that \( ca \equiv cb \pmod{cm} \).

4) Let \( a, b \in \mathbb{Z} \) such that \( a \equiv b \pmod{m} \). Prove that \( (a, m) = (b, m) \).

5) Let \( n \) be an odd integer not divisible by 3. Prove that \( n^2 \equiv 1 \pmod{24} \).

6) Find all incongruent solutions to the following congruences.
   a) \( 12x \equiv 16 \pmod{32} \)
   b) \( 623x \equiv 511 \pmod{679} \)
   c) \( 481x \equiv 627 \pmod{703} \)

7) How difficult was this homework? How long did it take?