Math 115A Homework 7

1) a) Find the remainder of $79^{79}$ divided by 9.
    b) Find the remainder of $99^{999999}$ divided by 26.
    c) Find a multiplicative inverse of $4^{67}$ modulo 19.

2) a) Let $n$ be an integer not divisible by 3. Prove that $n^7 \equiv n \pmod{63}$.
    b) Let $n$ be an integer divisible by 9. Prove that $n^7 \equiv n \pmod{63}$.

3) a) Prove that if $n$ is an integer relatively prime to 72, then $n^{12} \equiv 1 \pmod{72}$.
    b) Find the largest integer $m$ such that $n^{12} \equiv 1 \pmod{m}$ for all integers $n$ relatively prime to $m$. (Note: by part (a) this number must be at least 72).

4) Let $m$ and $n$ be relatively prime positive integers. Prove that $m^{\phi(n)} + n^{\phi(m)} \equiv 1 \pmod{mn}$.

5) Find $\phi(n)$ for each value of $n$ below.
    a) 64
    b) 105
    c) 15!

6) Are there any positive integers $n$ for which $\phi(n) = n/4$? Why or why not?

7) Let $m, n$ be positive integers such that $m|n$. Show that $\phi(m)|\phi(n)$. Is the converse true (i.e. is it true that if $\phi(m)|\phi(n)$ then $m|n$)?

8) How difficult was this homework? How long did it take?