Math 250A: Reading and Concepts for the week of October 11th and October 18th

- Finishing semidirect products. Review the notion of center and class equation.
- Group actions (we won’t prove thee orbit stabilizer lemma, and if you have never seen it before you may want to read its proof)
- Sylow theorems, proof of first Sylow theorem and a useful tool to prove the rest. Think about or look up proof of the following: an abelian group whose order is divisible by a prime \( p \) has an element of order \( p \). I will assume you are comfortable with the conjugation action.
- Using Sylow theorems in examples, motivating the study of fields and Galois theory.

General reading note: at this point we are soon transitioning into the study of fields: specifically, roots of polynomials, algebraic extensions, finite fields, and Galois theory to name some topics. I will assume that you are very familiar (and may do a review lecture) with the following concepts/results from the get-go (\( F \) always denotes a field):

- fields, field characteristic, field extension, simple field extension, degree of a field extension, finite extensions, algebraic vs. transcendental elements, minimal polynomial of an algebraic element,
- \( F(\alpha) \cong F[x]/(p(x)) \) where \( p(x) \) is an irreducible polynomial in \( F[x] \) and \( \alpha \) is a root of \( p(x) \); this tells us how to construct a field containing a zero of a given polynomial over \( F \)
- Irreducibility criteria for polynomials: see section 2.5
- A polynomial \( p(x) \in F[x] \) can have at most \( \deg(p) \) zeros in \( F \)
- A field has characteristic 0 or \( p \) for some prime \( p \)
- A finite multiplicative subgroup of a field is cyclic
- \( \alpha \) is algebraic over \( F \) if and only if the degree of \( \alpha \) over \( F \) is finite
- Finite extensions are algebraic
- Tower theorem: If \( E \) is a finite extension of \( F \) and \( F \) is a finite extension of \( K \) then \( [E : K] = [E : F][F : K] \)