Math 250A: Reading and Concepts for Lectures 1-4

General reading note: If you are not comfortable with undergraduate material defining groups, cyclic groups, homomorphisms, and cosets, as well as some classical examples like $S_n$ and $D_{2n}$, please read up on it. Also, for a summary of things you need to know about cyclic groups, see Arthur Ogus’s nice concise notes on our course website. To refresh your comfort with permutation groups, it might help also to look over George Bergman’s notes on the proof that $A_n$ is simple for $n \geq 5$ (also posted on the course website).

The lectures this week will be planned roughly as follows (it may very well take longer to cover than planned):

- Lectures 1 and 2: Quotient groups and isomorphism theorems, a bit on exact sequences. I assume you know what the following are: groups, (normal) subgroups, quotient groups, index, homomorphism, image, kernel, the canonical morphism $G \to G/H$ where $H \triangleleft G$, centralizer and normalizer (I may remind you briefly what some of these are).
  
  It will be useful to think about the following exercises:
  
  – If $K$ is any subgroup of $G$ containing $H$ and such that $H$ is normal in $K$, then $K$ is contained in $N_H$, the normalizer of $H$ in $G$.
  
  – If $K$ is a subgroup of $N_H$, then $KH$ is a group and $H$ is normal in $KH$.
  
  – The normalizer of $H$ is the largest subgroup of $G$ in which $H$ is normal.

- Lectures 3 and 4: Composition series, beginning Jordan-Hölder theorem, abelian and cyclic towers (series). I assume you know what simple, abelian, and cyclic groups are. You should be comfortable with the isomorphism theorems covered last time, and should convince yourself of the following:
  
  If $G_1, H_1$ are normal subgroups of $G$, then $G_1 H_1$ is a normal subgroup of $G$. Also, if $H$ is a normal subgroup of $G$ and $K$ is a normal subgroup of $G$ contained in $H$, then $H/K$ is a normal subgroup of $G/K$. Finally, the intersection of normal subgroups is normal.