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Clasps formulas in Type B_2/C_2

Starting pt $U_q(\mathfrak{sl}_2)$ has a natural rep. V.

$\mathrm{Fund}(U_q(\mathfrak{sl}_2)) = \langle V \rangle_{\otimes}$

has a diagrammatic presentation

$$| = \mathrm{id}_V \qquad V\otimes V \xrightarrow{} \mathrm{triv} = \cap \qquad \mathrm{triv} \xrightarrow{} V\otimes V = \cup$$

Relations $\quad = | = \quad , \quad \bigcirc = -[2]_q$

$$\mathrm{Rep}_{f.d}(U_q(\mathfrak{sl}_2)) = \mathrm{Kar}(\mathrm{Fund}(U_q(\mathfrak{sl}_2)))$$

Facts from rep. theory of $U_q(\mathfrak{sl}_2)$

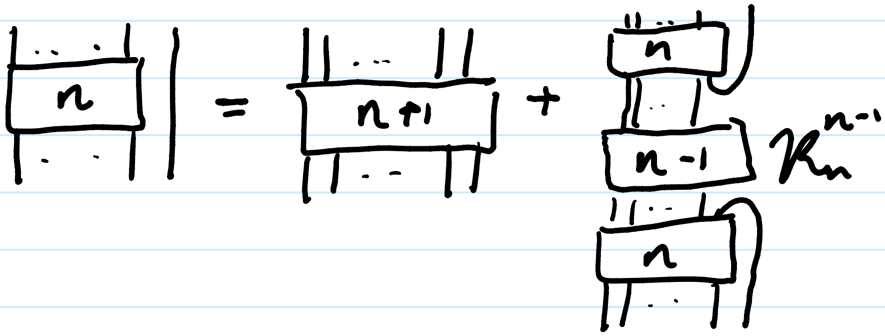
$$\mathbb{Z}_{\geq 0} \longleftrightarrow \text{f.d. reps of } U_q(\mathfrak{sl}_2)$$
$$n \longmapsto V_n.$$

$V_n \overset{\oplus}{\subseteq} V^{\otimes n}$, w/mult. 1.

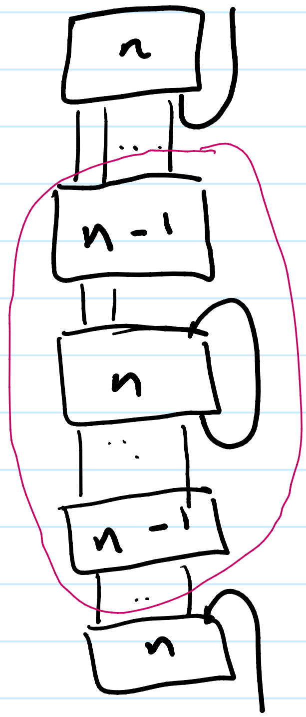
$V_n \not\subseteq V^{\otimes k}$ for $k < n$

$V_n \otimes V = V_{n+1} \oplus V_{n-1}$, $V_{-1} = 0$

We deduce the Jones-Wenzl recursion:

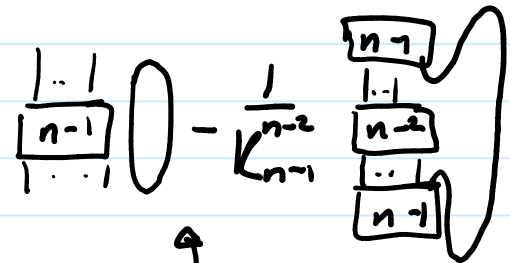


<u>Goal</u> Compute K_n^{n-1}:



Use JW recursion

$\rightsquigarrow$



K_n^{n-1} = coeff of id

$= -[2] - \dfrac{1}{K_{n-1}^{n-2}}$

<u>Solution</u> $K_1^0 = -[2]$,

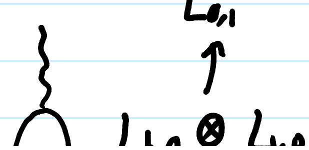
$$K_n^{n-1} = \frac{-[n+1]}{[n]}$$

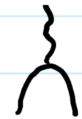
What about C_2?

$sp_4 \circlearrowright$ $L_{1,0}$ ← natural rep

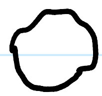
$L_{0,1}$ ← $\Lambda^2(\text{nat})/\text{triv}$

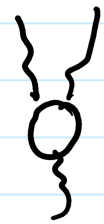
(Kuperberg '97) $| = id_{L_{1,0}}$ $\} = id_{L_{0,1}}$

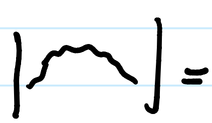
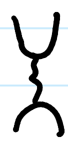
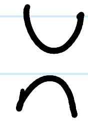


(Kuperberg '97) $| = id_{L_{1,0}}$ $\} = id_{L_{0,1}}$  $L_{1,0} \otimes L_{1,0}$

<u>Relations</u> $v = q^{1/2}$  $= \frac{-[6]_v[2]_v}{[3]_v}$

 $= \frac{[6]_v[5]_v}{[3]_v[2]_v}$

 $= -[2]_v^2 \}$,  = zero,  = zero

 =  + $||$ − 

Do a similar (but much more complicated!) analysis to find

$$K^{n+2,m-1}_{n,m}$$

to get 7 (!!!) recursions that can be solved and agree with

<u>Clasp Conjecture</u> (Elias)

$$K^{\lambda+\mu}_{\lambda} = \prod_{\alpha \in \Phi_+(\mu)} \frac{[(\alpha^\vee, \lambda+\rho)]_{v \text{ or } v^2}}{[(\alpha^\vee, \lambda+\rho+\mu)]_{v \text{ or } v^2}}$$

Known up to sl_4 and now sp_4 too.