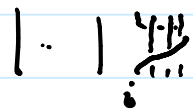
Ben Elias

quinta-feira, 20 de junho de 2019 09:54

$W_{fin} = S_n$, $\mathbb{H}_{fin}$ = Hecke algebra

$J \subseteq \mathbb{H}_{fin}$, large comm. subalgebra,
gen-d by multiplicative JM elements

JM elements: $Br_{fin} \to \mathbb{H}_{fin}$

$j_i =$ 

The center of $\mathbb{H}_{fin}$, Z_{fin} is spanned by symm. poly in JM elements

Ex $ft_n := e_n(j)$ - the full twist

$t := e_1(j)$

Categorification

$\mathcal{H}_{fin} := K^b(SBim_{fin})$

Indecomposables: $\{B_w\}$, $w \in W_{fin}$

$[B_w] = b_w$ = KL elements in $\mathbb{H}_{fin}$

To cat-fy the braid group:

Rouquier complexes $\beta \in Br_{fin}$ $Rouq(\beta) \in \mathcal{H}_{fin}$

Rouquier complexes | $\in Z_{fin}$ Rouq(j) $\in Z_{fin}$

So we get subcat-y $\mathcal{J} \subseteq \mathcal{H}_{fin}$ gen'd by Rouq(j_i) =: J_i

Z_{fin} : Drinfeld center
(Objects are $Z \in \mathcal{H}$ with nat. iso $\cdot \otimes Z \Rightarrow Z \otimes \cdot$)

Note: Drinfeld center of a Δ'd braided monoidal cat-y is Δ'd

GNR conjecture: $Z_{fin} \xrightarrow{\sim}$ "$D^b Coh(Hilb_n)$"
Gorsky, Negut, Rasmussen

$FT_n \longmapsto \mathcal{O}(1)$

$$\begin{tikzcd}
J_i \arrow[d, leftrightarrow] & \mathcal{J} \arrow[d, "\cong"] & & Z_{fin} \arrow[ll] \arrow[d] \\
\mathcal{L}_i & D^b Coh FHilb_n & & \text{"}D^b Coh(Hilb_n)\text{"} \arrow[ll, "\pi^*"']
\end{tikzcd}$$

taut. bundles Eigenvalues

$T \in D^b Coh(Hilb_n)$ taut. n. plane bundle
$\det T = \mathcal{O}(1)$

On FHilb, T has a filtration by line bundles $\mathcal{L}_i$

$\Rightarrow T$ categorifies $t = e_1(j)$.

GNR says: To prove conjecture, find $T \in Z_{fin}$

$$\begin{tikzcd}
\mathbb{H}_{ext} \arrow[ddd] & & \Lambda_{wt} \arrow[ll] \arrow[dl] \arrow[ddd] & \mathcal{Z}_{ext} \arrow[l] \arrow[ddd] & {[\mathrm{Rep}\,\mathfrak{gl}_n]} \arrow[l, "\sim"'] \\
& Br_{ext} \arrow[ul] \arrow[d] & & & \\
& Br_{fin} \arrow[dl] & & & \\
\mathbb{H}_{fin} \arrow[rr, dashed, bend right] & & \mathcal{J} \arrow[ll] \arrow[ul] \arrow[r, dashed, bend right] & \mathcal{Z}_{fin} \arrow[l] &
\end{tikzcd}$$

Note: $\mathbb{H}_{fin}$ has nondeg inner product $(-,-)|_{\mathcal{J}}$, $(-,-)|_{\mathcal{Z}}$ is still nondeg.

$\Rightarrow$ $\iota : \mathcal{J} \hookrightarrow \mathbb{H}$ has an adjoint

$\iota^* : \mathbb{H} \to \mathcal{J}$

$\iota^* : \mathbb{H} \to \mathcal{Z}$

<u>**Note**</u> $HHH(\beta)$ "=" $\mathrm{Hom}(\mathbb{1}, \mathrm{Rav}_q(\beta))$

$\downarrow$

$(\mathbb{1}, \beta)_{\mathbb{H}} = (\mathbb{1}, \iota^*)_{\mathcal{Z}}$

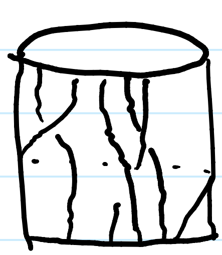
So, enough to work on $\mathcal{Z}$ to compute HHH.

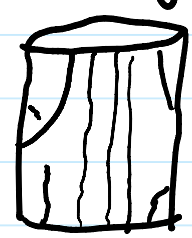
- Extended Affine story

$$W_{ext} = \Lambda_{wt, \mathfrak{gl}_n} \rtimes W_{fin}$$

$\mathbb{H}_{ext}$ has Λ_{wt} as large comm. subalg

Br_{ext} = cylindrical braid group

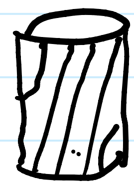


f_0 =  $\langle Br_{fin}, f_0 \rangle = Br_{aff}$

$\uparrow$

All these have winding # = 0

All these have winding # $=0$.

$\varpi =$  $\leftarrow$ Winding # $=1$

$Br_{ext} = \langle Br_{aff}, \varpi \rangle$

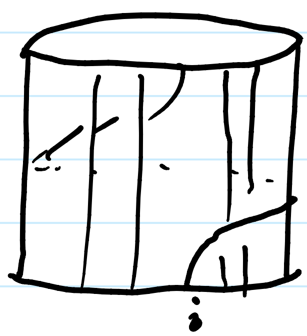
<u>Note</u> $Z(Br_{ext}) = \langle \varpi^n \rangle$

(Imposing $\varpi^n = 1$ we get Br^{ab}_{ext}, but we <u>won't</u> do this)

<u>Note</u> $Br_{ext} = Br_{aff} \rtimes \mathbb{Z}$ $\leftarrow$ power of ϖ.

Conj. by ϖ is Dynkin diagram rotation

Lattice $\Lambda_{wt} \subseteq Br_{ext}$. Let

$y_i =$ 

$\langle y_1, \dots, y_n \rangle = \mathbb{Z}^n$

Think: $y_i = (0 \cdots 0\, 1 \cdots 0)$

<u>Note</u> $n!$ diff. lattices, b/c we could order y_i's differently

[illegible]

$\mathcal{H}_{ext} = Br_{ext}/$ quad. reln

<u>Categorification</u> We have $SBim_{aff}$, $\mathcal{H}_{aff} = K^b(SBim_{aff})$.

To get $SBim_{ext}$, $\mathcal{H}_{ext}$, formally add Ω, invertible, s.t.

$$\Omega B_s \Omega^{-1} \cong B_{rot(s)}$$

(No homs btw distinct powers of Ω)

Still have $\mathrm{Rouq}(\beta)$ for $\beta \in Br_{ext}$

$\mathrm{Rouq}(\beta)$ for $\beta \in \Lambda_{wt}$ are called <u>Wakimoto</u> complexes

$$\begin{tikzcd}
\mathcal{H}_{ext} \arrow[dd] & & \mathrm{Wak} \arrow[ll] \arrow[dl] & \mathcal{Z}_{ext} \arrow[l] \arrow[dd] & \mathrm{Rep}\,\mathfrak{gl}_n \arrow[l] \\
& \mathrm{Rouq}_{ext} \arrow[ul] \arrow[d] & & & \\
\mathcal{H}_{fn} & \mathrm{Rouq}_{fn} & \mathcal{J} \arrow[ll] \arrow[l] \arrow[d, leftrightarrow, "?"] & \mathcal{Z}_{fn} \arrow[l] \arrow[d, leftrightarrow, "?"] & \\
& & \text{"}D^b\mathrm{Coh}(\mathrm{wFHilb})\text{"} & \text{"}D^b\mathrm{Coh}\,\mathrm{Hilb}_n\text{"} &
\end{tikzcd}$$

Vertical arrows: Flattening

<u>Thm</u> $\exists$ categorification

$$b : SBim_{ext} \longrightarrow \mathcal{H}_{fn}$$

$b : \mathrm{SBim}_{ext} \longrightarrow \mathcal{H}_{fn}$

<u>Want/Conj</u> $\quad b : \mathcal{H}_{ext} \longrightarrow \mathcal{H}_{fn}$

If it does exist, $Y_i \longmapsto J_i$
Wakimoto complex

Note: flattening $\mathcal{O}^\circ$ get full twist, that's why we need $\mathfrak{gl}_n$ and <u>not</u> $\mathfrak{sl}_n$

How does flattening help GNR?

$Z_{ext} = \Lambda_{wt}^{\mathfrak{S}_n} = [\mathrm{Rep}\,\mathfrak{gl}_n]$
$\mathrm{ch}V \leftarrow V$

Categorified by Gaitsgory.

$\mathrm{Rep}\,\mathfrak{gl}_n \xrightarrow{\mathcal{Y}} \mathcal{Z}_{ext}$ (Uses Geom. Satake) & Nearby Cycles

Fully faithful in deg 0

$\mathrm{Im}\,\mathcal{Y}$ = Gaitsgory's central complexes

$V = \bigoplus_{\lambda \in \Lambda_{wt}} V[\lambda] \Rightarrow \mathcal{Y}(V)$ has a filtration w/ subquotients Wakimoto cplxes

GNR wanted: $\mathcal{T}$ w/ filtration s.t. J_i appears once

Affine version: $\mathcal{V}$ with filt. s.t. Y_i appears once

Affine version: ... it appears once

<u>Great candidate</u> : $\mathcal{V} = \mathcal{Y}(V_{standard})$

<u>Conjecture</u> : $\mathcal{T} = b\mathcal{Y}(V_{st})$

<u>BS</u> Explicitly computes $\mathcal{Y}(V_{st})$, reconstructing it by hand.

<u>Answer</u> $n=2$

$\overset{\infty}{\underset{0\quad 1}{\bullet\!-\!\!-\!\!-\!\bullet}}$

Conjugate Wakimoto

Wakimoto

$$\Omega\left(\mathbb{1}(-1) \to \begin{matrix} B_0 \\ \oplus \\ \underline{B_1} \end{matrix} \to \mathbb{1}(1)\right)$$

<u>Properties</u> • $n!$ Wakimoto filtrations
• They're perverse: grading shift = hom. shift!

• Central

$$\Omega\left(\cancel{\mathbb{1}} \begin{matrix} \cancel{B_0} \\ B_1 \end{matrix} \cancel{\mathbb{1}}\right)\cancel{B_0} = \Omega B_1 B_0$$

Same answer multiplying on other side

Main thm: Explicit comp. of $\mathcal{V} \otimes B_{\omega_I}$.