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(jt. w/ Gorsky-Wedrich)

① A-algebra $Z(A) \circlearrowleft A/[A,A]$ $\swarrow$ v.s. only

center cocenter

If A is s/s f.d. $\Rightarrow Z(A) \to A \to A/[A,A]$ is iso
char $=0$

$\mathbb{H}_n$ - Hecke alg. for $S_n/\mathbb{Q}(q)$ is semisimple and

$$Z(\mathbb{H}_n) \cong \text{deg } n \text{ symm. functions}$$
$$\cong \mathbb{H}_n/[\mathbb{H}_n,\mathbb{H}_n]$$

Conj [GNR] $\exists$ pair of adjoint functors (F,G)

$$K^b(SBim_n) \underset{F}{\overset{G}{\rightleftarrows}} \text{"}D^b(Hilb^n \mathbb{C}^2)\text{"}$$

categorifying relationship betw $\mathbb{H}_n$ and $Z(\mathbb{H}_n)$

$$\mathbb{H}_n \underset{\text{inclusion}}{\overset{\text{projection}}{\rightleftarrows}} Z(\mathbb{H}_n)$$

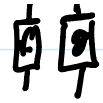
Goal: Introduce dg version of cat'fd (co)center & apply to $K^b(SBim_n)$. (Studied by Ben-Zvi–Nadler).

② Horizontal trace (cat'fd cocenter)

$\mathcal{C}$ = k-linear cat-y, $\oplus$, $\otimes$.

Morphisms: Depicted using diagrams  denotes $f: X \to Y$

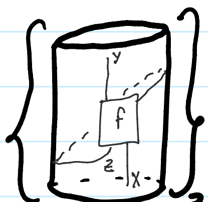
$f \circ g =$  $f \otimes g =$ 

$f\circ g =$  $f\otimes g =$  1_X

Roughly speaking, horizontal trace draws diagram in a cylinder

$hTr(\mathcal{C})$

Objects: Same as $\mathcal{C}$

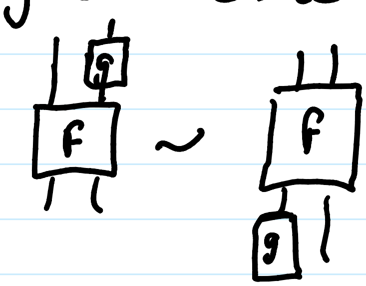
Diagrams $\Big\{$  $\Big\}_{Z\in\mathcal{C}} \simeq \bigoplus_{Z\in\mathcal{C}} \mathrm{Hom}(Z\otimes X, Y\otimes Z)$

w/ relations

$(Id\otimes g)\circ f \sim f\circ(g\otimes id)$

$f: Z\otimes X \longrightarrow Y\otimes Z'$

$g: Z\longrightarrow Z'$, ie

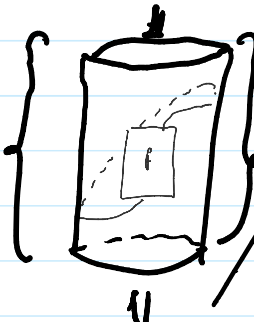


Claim (Slight lie)

$\overline{X\otimes Y} \simeq \overline{Y\otimes X}$ in hTr

(from now on, $\overline{X}$ denotes an object in hTr)

Invertible when Y is dualizable

$\mathrm{End}_{hTr(\mathcal{C})}(\mathbb{1}) = \Big\{$  $\Big\}\Big/\sim \;\cong \bigoplus_{Z\in\mathcal{C}} \mathrm{End}_{\mathcal{C}}(Z)\Big/\sim$

$\uparrow$

vertical trace

vertical trace
$vTr(\mathcal{C})$
$= HH_0(\mathcal{C})$

$f \circ g = g \circ f$

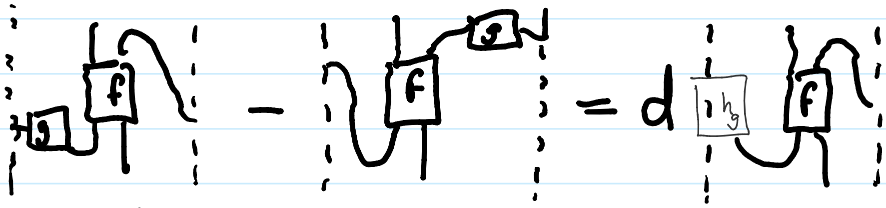
$hTr(\mathcal{C}) = \text{Cat-y.}$ $\qquad vTr(\mathcal{C}) = \text{algebra}$

E.g. $\mathcal{C} = \mathbb{S}\mathrm{Bim}_n$ or $\underbrace{K^b(\mathbb{S}\mathrm{Bim}_n)}$

Want a version of cocenter where relations are only satisfied up to homotopy.

⑤ Derived Versions

Mantra Replace relations by homotopies "living at the seam"



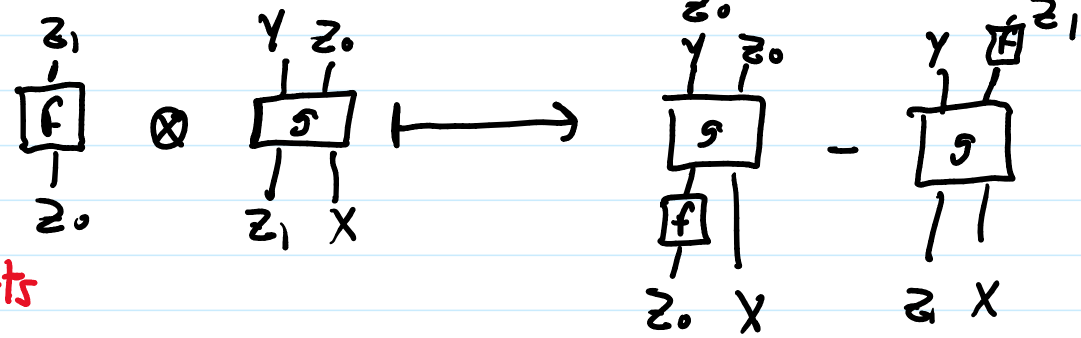
Def $hTr^{dg}(\mathcal{C})$ has objects $Ob(\mathcal{C})$, morphisms $\overline{X} \to \overline{Y}$

cohomological degree $\to$
$$\bigoplus_{r=0}^{\infty} \bigoplus_{Z_0,\dots,Z_r \in \mathcal{C}} \mathrm{Hom}(Z_0,Z_1) \otimes_k \cdots \otimes_k \mathrm{Hom}(Z_{r-1},Z_r) \otimes_k \mathrm{Hom}(Z_r \otimes X, Y \otimes Z_0)[r]$$

w/ cyclic bar differential, that starts as follows

$$\bigoplus_{Z_0,Z_1} \mathrm{Hom}(Z_0,Z_1)\otimes \mathrm{Hom}(Z_1X, YZ_0) \longrightarrow \bigoplus_{Z_0} \mathrm{Hom}(Z_0X, YZ_0)$$

Morphisms don't just slide thru the seam! It costs a homotopy



<u>Rmk</u> $H(\mathrm{End}_{hTr^{dg}(\mathcal{C})}(\mathbb{1})) \cong HH_0(\mathcal{C})$, as algebras

<u>E.g.</u> $\mathcal{C} = \{\mathbb{1}\}$, $A = \mathrm{End}_{\mathcal{C}}(\mathbb{1})$ commutative (Eckmann-Hilton)

$HH_\bullet(A)$ is an algebra via shuffle product

④ $hTr^{dg}(\mathrm{SBim}_n)$

<u>Thm</u> (Gorsky-H-Wedrich)

Homotopies that slide x_i around cylinder (see below)

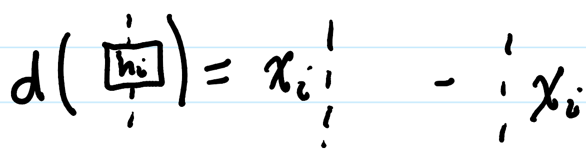
$HH\deg = 1$

$$HH_\bullet(\mathrm{SBim}_n) \cong (\mathbb{Q}[x_1,\dots,x_n]\otimes \Lambda[\theta_1,\dots,\theta_n]) \rtimes \mathbb{Q}[S_n]$$

$$\cong R\mathrm{End}(\text{Springer sheaf})$$

$$\cong H^\bullet(\text{Steinberg}) \;(¿?)$$

Found!



<u>Conj</u> $hTr^{dg}(\mathrm{SBim}) =$ perf dg-mod. over

$$\overline{X} \longmapsto \mathrm{Hom}_{hTr^{dg}}(\mathbb{1}, \overline{X}) \circlearrowleft \mathrm{End}(\mathbb{1})$$

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Note Underived statements are known

$HH_0(\mathbb{S}Bim_n) \cong \mathbb{Q}[x_1, \ldots, x_n] \rtimes S_n$ (Elias-Lauda)

$hTr(\mathbb{S}Bim_n) \cong$ proj-ve modules over (Queffelec-Rose)

Idea of proof $HH_\bullet(\mathcal{C}) \cong HH_\bullet(Ch^{dg}(\mathcal{C}))$

Can be calculated via semi-orth. dec

$Ch(\mathcal{C})$ generated by $\mathcal{B}_i \subseteq \mathcal{C}$ with $hom(X, Y) \simeq 0$ unless $X \in \mathcal{B}_i$, $Y \in \mathcal{B}_j$, $i \leq j$.

For $\mathbb{S}Bim$, $Ch^b(\mathbb{S}Bim)$ are gen-d by $\bigoplus_{w \in S_n} Rouq(w)$

semiorth w.r.t Bruhat

$\Rightarrow HH_\bullet(\mathbb{S}Bim_n) \simeq \bigoplus_w HH_\bullet(\langle Rou_w \rangle)$

$\langle Rouq_w \rangle \cong R\text{-grmod}$

So at least vect sp. dec. is clear.