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A combinatorial description of some reps of dAHA

Etingof-Freund-Ma Functor

$$F_{n,p,q} : GL_N\text{-reps} \longrightarrow \text{dAHA-rep } H_n(K_1,K_2)$$

$N = p+q$

$$M \longmapsto F(M) \subseteq M \otimes V^{\otimes n}, \quad V = \mathbb{C}^N$$

dAHA via generators & Relations

$$W_{B_n} = S_n \ltimes (\mathbb{Z}/2\mathbb{Z})^n = \langle s_1, \dots, s_{n-1}, \gamma_n \mid \text{Coxeter relations} \rangle$$

$$H_n(K_1,K_2) = \mathbb{C}W \ltimes \mathbb{C}[y_1, \dots, y_n] \text{ w/relations}$$

$$s_i y_i - y_{i+1} s_i = K_1$$

$$\gamma_n y_n + y_n \gamma_n = K_2$$

So, we need to give action on $F(M)$

On $m \otimes v_1 \otimes \cdots \otimes v_n$:

- s_i permutes v_i and v_{i+1}
- γ_n acts by $\begin{pmatrix} I_p & \\ & -I_q \end{pmatrix} v_n$
- y_i acts by $Y_i := -\sum_{k,j} (E^j_k)_0 \otimes (E^k_j)_i + \cancel{\frac{p-q}{2}} - \mu N \gamma_i$

$$+ \frac{1}{2} \sum_{t>i} s_{it} - \frac{1}{2} \sum_{t<i} s_{it} + \frac{1}{2} \sum_{i \neq t} s_{it} \gamma_i \gamma_t$$

$$+\frac{1}{2}\sum_{t>i} s_{it} - \frac{1}{2}\sum_{t<i} s_{it} + \frac{1}{2}\sum_{i\neq t} s_{it} y_i y_t$$

<u>Claim</u> M f.d. irrep $\Rightarrow$ $\mathcal{F}(M)$ is Y-semisimple

M f.d. irrep $\Rightarrow$ $M = \det^{-a} \otimes V^{\lambda}$, V^{λ} Schur module

Set $K = GL_p \times GL_q \subseteq GL_N$, $\mathfrak{k} = \mathfrak{gl}_p \oplus \mathfrak{gl}_q = \mathrm{Lie}(K)$

$$\mathfrak{k}_0 = \{X \in \mathfrak{k} \mid \mathrm{tr}(X) = 0\}$$

$$\chi : \mathfrak{k} \to \mathbb{C}, \quad \chi\begin{pmatrix} \overbrace{A}^{p} & \overbrace{0}^{q} \\ 0 & B \end{pmatrix} = q\,\mathrm{tr}A - p\,\mathrm{tr}B$$

$\mu \in \mathbb{C}$:

$$F_{n,p,\mu}(M) = (M \otimes V^{\otimes n})^{\mathfrak{k}_0, \mu\chi}$$

$$\cong \mathrm{Hom}_{\mathfrak{k}_0}(\mathbb{1}_{\mu\chi}, M \otimes V^{\otimes n})$$

$$\cong \mathrm{Hom}_K(\mathbb{1}_{\theta}, M \otimes V^{\otimes n}), \text{ where}$$

Here we're in the case $M = V^{\lambda}$

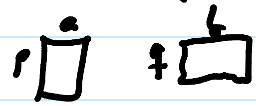
$$\mathbb{1}_{\theta} = \det^{\underbrace{\mu q + \frac{|\lambda| + n}{N}}_{a}} \boxtimes \det^{\underbrace{-\mu p + \frac{|\lambda| + n}{N}}_{b}}$$

$$s_{\lambda}(x_1, \dots, x_p, y_{p+1}, \dots, y_N) = \sum_{\mu \subseteq \lambda} c^{\lambda}_{\mu\nu} s_{\mu}(x_1, \dots, x_p) s_{\nu}(y_{p+1}, \dots, y_N)$$

where $c^{\lambda}_{\mu\nu}$ is st $s_{\mu} s_{\nu} = \sum_{\lambda} c^{\lambda}_{\mu\nu} s_{\lambda}$

then we get $S_{(a,p)} \times S_{(b,q)}$

then we get $s_{(a^p)} \times s_{(b^q)}$

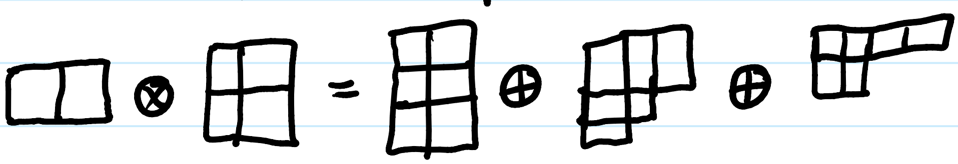


<u>Prop</u> (Okada) $c^{\lambda}_{(a^p)(b^q)} = 1$ or 0, and it is 1 iff λ satisfies

$$\lambda_i + \lambda_{p+q-i+1} = a+b \quad (i=1,\dots,p)$$

$p \le q$ $\qquad \lambda_p \geq \max(a,b)$

$$\lambda_i = b, \quad i = p+1,\dots,q$$



Another way to decompose $V^{\lambda} \otimes V^{\otimes n}$ is multiplying V^{λ} by □ n times.