6/19/19 UC-Davis talk

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Thm (GORS). Coprime $m, n > 0$:

$$\textsc{HOMFLY}_{a,q}(T_{m,n}) \quad \propto \quad \sum_j (-a^2)^j \langle \Lambda^j(\mathfrak{h}), [L_{m/n}]_q \rangle_{S_n}$$

where

$$\Lambda^j(\mathfrak{h}) = \text{Specht module of } \textit{jth hook partition}$$
$$[L_{m/n}]_q = q\text{-graded } S_n\text{-char of the f.d. irrep of } \mathfrak{H}_{m/n}(S_n)$$

Goal. Explain "why" by refining HOMFLY

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Weyl gp $W \curvearrowright \mathfrak{h}$. $\quad s_i \mapsto \beta_i \in \mathrm{Br}_W^+$

$\beta = \beta_{i_1}\beta_{i_2}\cdots\beta_{i_\ell} \in \mathrm{Br}_W^+$

$$\begin{array}{c} G\backslash Y_\beta \\ \pi_\beta \;\downarrow \\ G\backslash G \end{array} \qquad Y_\beta = \left\{ \begin{array}{l} (g, B_1, \ldots, B_\ell) \in G \times \mathcal{B} \times \cdots \times \mathcal{B} \\ \quad \mid gB_\ell g^{-1} \xrightarrow{s_{i_1}} B_1 \xrightarrow{s_{i_2}} \cdots \xrightarrow{s_{i_\ell}} B_\ell \end{array} \right\}$$

π_β depends only on conjugacy class of β

Ex. $G\backslash Y_1 \xrightarrow{\pi_1} G\backslash G$ is the Springer map

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Steinberg-like stack $\quad \mathrm{St}_\beta = G\backslash Y_1 \underset{G\backslash G}{\times} G\backslash Y_\beta$

$$\begin{array}{ccccc} & & G\backslash Y_1 \underset{G\backslash G}{\times} G\backslash Y_1 \underset{G\backslash G}{\times} G\backslash Y_\beta & & \\ & \swarrow & \downarrow & \searrow & \\ \mathrm{St}_1 & & \mathrm{St}_\beta & & \mathrm{St}_\beta \end{array} \qquad \mathrm{H}_*(\mathrm{St}_1) \curvearrowright \mathrm{H}_*(\mathrm{St}_\beta)$$

$$\underbrace{\mathrm{H}_*(\mathrm{St}_\beta)}_{\text{v. s.}} \xrightarrow{\text{Rouquier-ify}} \underbrace{\mathrm{Chr}(\mathrm{St}_\beta)}_{\text{"chromatographic" complex}}$$

external degree
internal bigrading from $\mathrm{gr}_*^{\mathrm{W}}\,\mathrm{H}^*(\mathrm{St}_1)$

Markov char. $\quad (q, t)$-bigraded virtual v.s.

$$\mathbb{M}_{q,t}(\beta) = \bigoplus_{i,k} q^i t^k\, \mathrm{H}_k(\mathrm{Chr}(\mathrm{St}_\beta)^{i,i})$$

$$\mathbb{M}_1 = \mathbb{C}[W] \ltimes \mathrm{Sym}_q^*(\mathfrak{h}) \curvearrowright \mathbb{M}_\beta$$

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Thm (T). $\quad \beta$ arbitrary:

$$\textstyle\sum_j (a^2 t)^j \langle \Lambda^j(\mathfrak{h}), \mathbb{M}_{q,t}(\beta) \rangle_W \quad \propto \quad \mathrm{HHH}_{a,q,t}^{*,*,*}(\beta)$$

Thm (T). $\quad \beta = \mathrm{FT}^{m/n}$:

$$\mathbb{M}_{q,t}(\beta)|_{t=-1} \propto \bigoplus_{\phi \in \hat{W}} q^{\frac{m}{n} c_\phi} D_\phi(e^{2\pi i \frac{m}{n}})\phi \otimes \mathrm{Sym}_q^*(\mathfrak{h})$$

where $\begin{cases} c_\phi & = \text{content number of } \phi \\ D_\phi & = \text{generic deg. poly. of } \phi \end{cases}$

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RCA $\mathfrak{H}_{m/n}(W) = (\mathrm{Sym}^*(\mathfrak{h}) \rtimes \mathbb{C}[W] \ltimes \mathrm{Sym}^*(\mathfrak{h}^\vee))/I_{m/n}$

Category O: $\left.\begin{matrix} \text{standards} & \Delta_{m/n}(\phi) \\ \text{simples} & L_{m/n}(\phi) \end{matrix}\right\}$ for $\phi \in \hat{W}$

Each module V has a q-graded W-character $[V]_q$, e.g.,

$$[\Delta_{m/n}(\phi)]_q = q^{\frac{1}{2}\,\mathrm{rk}(\mathfrak{h}) - \frac{m}{n} c(\phi)} \phi \otimes \mathrm{Sym}^*_q(\mathfrak{h})$$

<u>Cor (T).</u> $\beta = \mathrm{FT}^{m/n}$:

$$\mathbb{M}_{q,t}(\beta)|_{t=-1} \simeq \bigoplus_\phi D_\phi(e^{2\pi i \frac{m}{n}})[\Delta_{m/n}(\phi)]_q$$

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Special case where we know what the $D_\phi(e^{2\pi i \frac{m}{n}})$ "mean"

<u>Cor (GGOR).</u> Assume $\frac{m}{n} \in \frac{1}{d_i}\mathbb{Z}$ for $\overbrace{\text{only one } d_i}^{\text{degree of } W}$:

$$\mathbb{M}_{q,t}(\mathrm{FT}^{m/n})|_{t=-1} = \begin{cases} [L_{m/n}(1)]_q + [L_{m/n}(\mathfrak{h})]_q & (W,n) = (E_8, 15), (H_4, 15) \\ [L_{m/n}(1)]_q & \text{else} \end{cases}$$

<u>Pf.</u> $D_\phi(e^{2\pi i \frac{m}{n}})$ are structure consts. for $\mathbb{H}_{q=e^{2\pi i m/n}}$-<u>mod</u>.

$$\mathrm{KZ} : \mathfrak{H}_{m/n}\text{-}\underline{\mathrm{mod}} \to \mathbb{H}_{q=e^{2\pi i m/n}}\text{-}\underline{\mathrm{mod}}$$

is "highest-wt cover", so can match blocks

Extra

<u>Df.</u> $\beta \in \mathrm{Br}_W^+$ is <u>algebraic</u> iff it's from $\mathrm{Spec}\,\mathbb{C}[[t]] \to \mathfrak{h} /\!\!/ W$

<u>Conj / Thm (T).</u> β is algebraic iff

$$\beta^n \sim \mathrm{FT}_{J_1} \cdots \mathrm{FT}_{J_\ell}$$

for some n and $J_1, \ldots, J_\ell \subseteq S$,

where FT_J = full twist for the parabolic $W_J \subseteq W$

<u>Conj (T).</u> β algebraic $\implies$ $\mathbb{M}_{q,t}(\beta)|_{t=-1}$ has positive coeffs.

... and "fourth grading" should be trivial