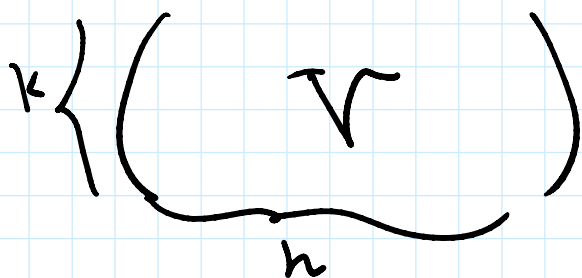


$Gr(k, n) = \text{Grassmann variety}$
 $= \{ k\text{-planes in } n\text{-dimensional space} \}$



$k \times n$ matrix of rank k

rows of $V =$ basis in a k -plane.

$GL(k)$ acts by multiplication on the left = base change



$v_1, \dots, v_n =$ columns of V

\rightsquigarrow repeat periodically

$v_1, v_2, v_3, \dots, v_n, v_1, v_2, \dots, v_n, \dots$
 $v_{i+n} = v_i$

We're interested in ranks of various subsets in this sequence

$$f(i) = \min \{ j : v_i \in \text{Span}(v_{i+1}, \dots, v_j) \}$$

$$i+n \geq f(i) \geq i \quad f: \mathbb{Z} \rightarrow \mathbb{Z}$$

$$v_{i+n} = v_i$$

\leftarrow = if $v_i = 0$

$$f(i+n) = f(i) + n \quad f(i) + \dots + f(i)$$

$$v_{1n} = v_1 \quad \dots \quad v_n = 0$$

Fact: f is a bijection! $f(i+n) = f(i) + n$
 $f(1) + \dots + f(n) = kn$
 affine permutation

$\overset{\circ}{\Pi}_f = \{ \text{all matrices } \begin{matrix} \nearrow \\ \searrow \end{matrix} \text{ with fixed function } f \}$
 positroid strata in $G(k, n)$

$G(k, n) = \bigsqcup_f \overset{\circ}{\Pi}_f$ when $\overset{\circ}{\Pi}_f$ is nonempty?

$\overset{\circ}{\Pi}_f$ are also enumerated by other combinatorial data:

- Rank matrix

$$r_{ij} = \text{rank} \{ v_1, \dots, v_j \}$$

- Pairs of permutations

(u, w) satisfying some

conditions $u \leq w$ in Bruhat order

$$w(1) \leq \dots \leq w(k)$$

$$w(k+1) \leq \dots \leq w(n)$$

- Juggling patterns...

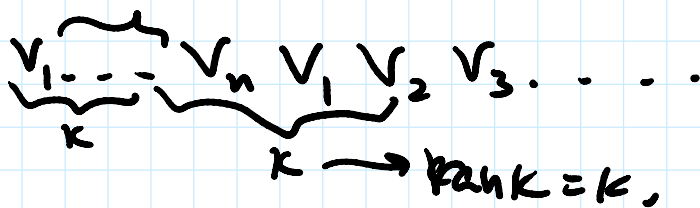
Thm [KLS] $\overset{\circ}{\Pi}_f = \text{projection of the Richardson variety } \chi_u^w \text{ under the map } \text{Flag}_n \rightarrow G(k, n)$

$f = ut \dots w$
 translation by k -th fundamental

the map $\text{Flag}_n \xrightarrow{0} G(k, n)$

Thm (KLS) The cohomology class of the (closure of) $\mathring{\Pi}_f =$ affine Stanley function for f

Ex Maximal positroid cell $\mathring{\Pi}_{k, n} \subset G(k, n)$
 $\text{rk}(v_i, \dots, v_{i+k-1}) = k$ for all i



Thm [GL] $H^*(\mathring{\Pi}_{k, n}) =$ rational q, t -Catalan number $C_{k, n}(q, t)$
 $q, t =$ homological grading + Hodge filtration $\text{GCD}(k, n) = 1$

Also related to link invariants

$f \leftrightarrow (u, w) \leftrightarrow \beta(u) \cdot \beta(w)^{-1} \in$ braid groups
 $H^*(\mathring{\Pi}_f)$ related to knot homology of $\beta(u) \cdot \beta(w)^{-1}$.

1/11 Erik

[KLS] Section 2-3 combinatorics [Daniel]

1/18 MLK day

1/25 Erik

[KLS] Section 4-5

1/25 Erik
2/1 Joseph (reprep
from last
quarter)
2/8 Daniel
2/15 Presidents day
2/22
3/1
3/8

[KLS] Section 4-5 (Yuzv
Ashkeigh)
geometry +
defn of point
+ properties
[KLS] Connection
to affine Stanley
[Wenzel]
[GL] 9, 11 - Catalan
→ knots
(Eugene)