Some more combinatorics related to k-Schur functions

Most of what I will discuss:
The <u>weak order</u> on $\tilde{S}_{k+1}$

Def. We say that $w \xrightarrow[k]{} v$ in $\tilde{S}_{k+1}$ if

$\ell(w)+1=\ell(v)$ and $s_i w = v$.

$w \leq v$ if $\exists u$: $uw=v$ and $\ell(u)+\ell(w)=\ell(v)$

Characterization on cores: directly from above.

$\tau \xrightarrow[k]{} \kappa$ if $\exists s_i$: $s_i \tau = \kappa$ and $\ell(\tau)+1=\ell(\kappa)$

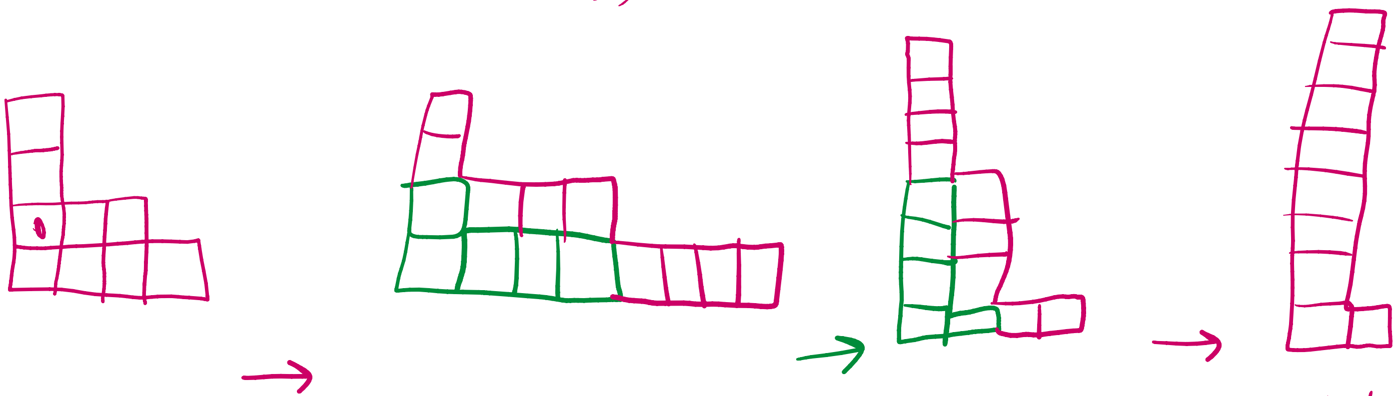
Characterization on bounded partitions:
... More complicated.

Want to say: $\lambda \xrightarrow[k]{} \mu$ if $\lambda \subset \mu$, $\lambda' \subset \mu'$, and $|\lambda|+1=|\mu|$

Actually: $\lambda \xrightarrow[k]{} \mu$ if $\lambda \subset \mu$, $\lambda^{\omega_k} \subset \mu^{\omega_k}$, and

Example of μ^{w_k}:

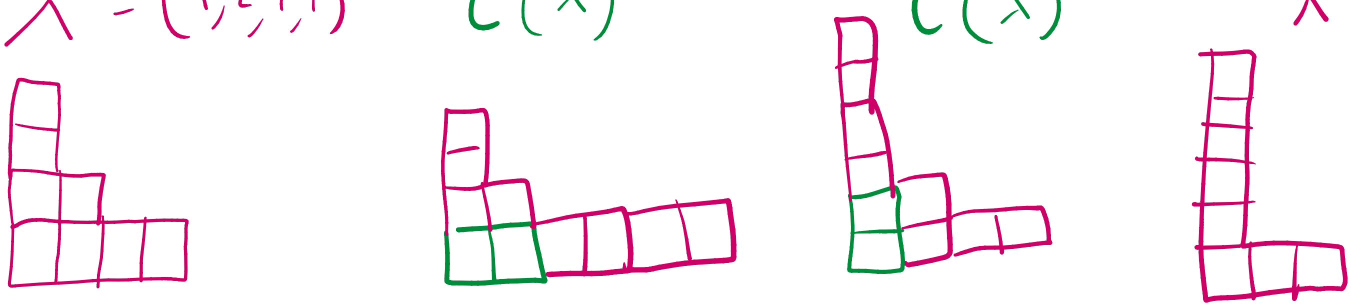
Let $\mu = (4,3,1,1)$ as a $k=4$-bounded partition.



$(2,1,1,1,1,1,1,1)$

$\mu = (4,3,1,1)$ $\quad c(\mu)$ $\quad c(\mu)'$ $\quad \mu^{w_k}$

$\lambda = (4,2,1,1)$ $\quad c(\lambda)$ $\quad c(\lambda)'$ $\quad \lambda^{w_k}$

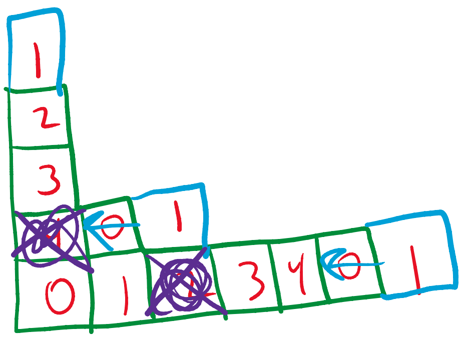


$(3,1,1,1,1,1)$

So here, $\lambda \subset \mu$, but $\lambda^{w_k} \not\subset \mu^{w_k}$.

Want: $\lambda \xrightarrow[k]{} \mu \iff \lambda \subset \mu,\ \lambda^{w_k} \subset \mu^{w_k},\ |\mu| = |\lambda| + 1.$

proof: $(\Rightarrow)$ $\quad \exists s_i : s_i \cdot C(\lambda) = C(\mu)$



Strong order on $\tilde{S}_{k+1}$:

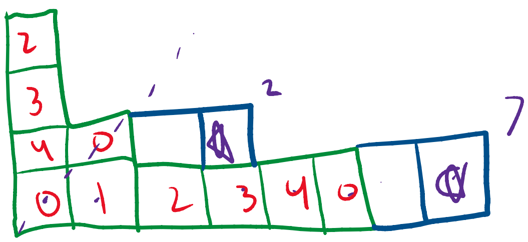
Def. We say that $w \underset{k}{\Rightarrow} v$ in $\tilde{S}_{k+1}$ if

$\ell(w)+1 = \ell(v)$ and $\exists t_{ij} : t_{ij} w = v$

Characterization on cores:

is the diagram a subset?

Related by <u>strong marked covers</u>



Remarkable property: if $\tau \underset{k}{\Rightarrow} \kappa$, then κ/τ consist of several translates of the same ribbon

Another way of viewing cores:

(Lam, Lapointe, Morse, Shimozono)