Meeting 1: Defining Lasagna Skein Module

Friday, January 13, 2023 1:01 PM

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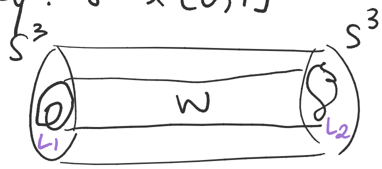
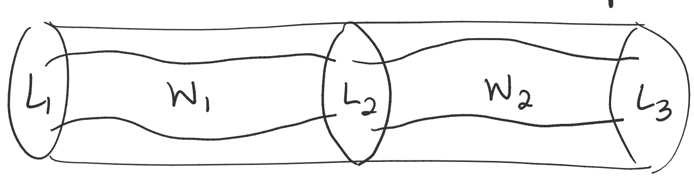
I) Recap on Kh

- L = oriented link in S^3
  $\rightsquigarrow$ $Kh(L)$ bigraded group
- Functoriality: $S^3 \times [0,1]$



W = smooth oriented surface in $S^3 \times [0,1]$
$\partial W = L_1 \sqcup L_2$
$\Phi_W : Kh(L_1) \longrightarrow Kh(L_2)$
linear map, preserves one grading
shifts another by $\pm \chi(W)$
minimal deg of surface

Prop: Invariant under smooth isotopies of W



$\varphi_{W_1 \cup W_2} = \varphi_{W_1} \circ \varphi_{W_2}$

[Jacobson, up to sign / $\mathbb{Z}_2$
Blanchet, Morrison, Capvan
Ehrig-Tubbenhauer-Wedrich (gl_N)]

Idea: perturb W s.t. t is a Morse fcn

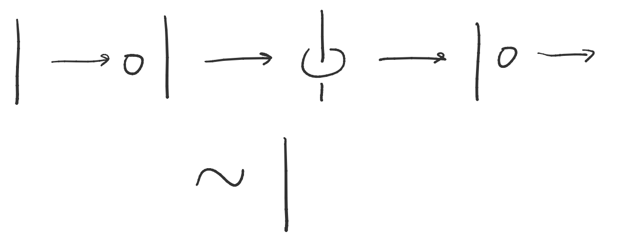
$\Rightarrow W$ is a "movie"

+ Riedemeister moves

- Construct Φ_W for any of the elementary moves & compose
- Isotopies of surfaces $\longleftrightarrow$ "movie moves"

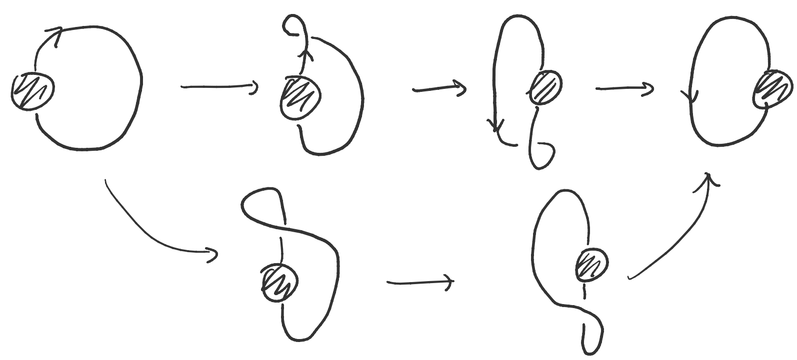
$\underbrace{\text{"movie moves"}}_{\text{24 of these}}$

e.g. a movie move



Isotopy in $S^3 \times [0,1]$ (as opposed to $\mathbb{R}^3 \times [0,1]$)

"sweep around" move

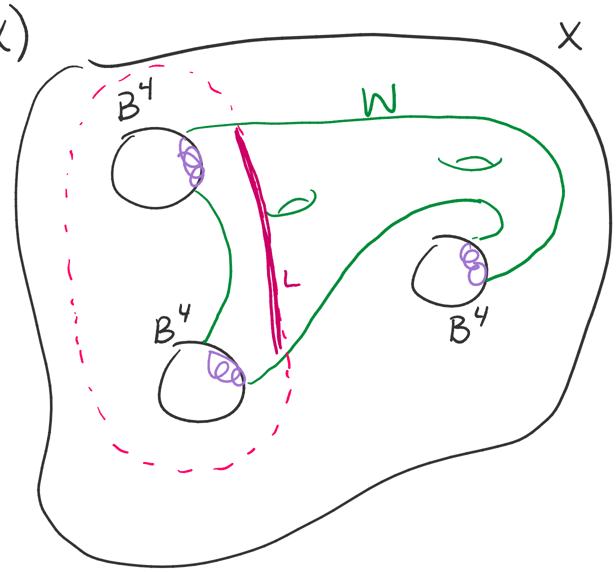


two surfaces are the same

Thm: (MWW) φ_W are same

X = smooth 4-mfld

def: lasagne skein module for $S_0^2(X)$



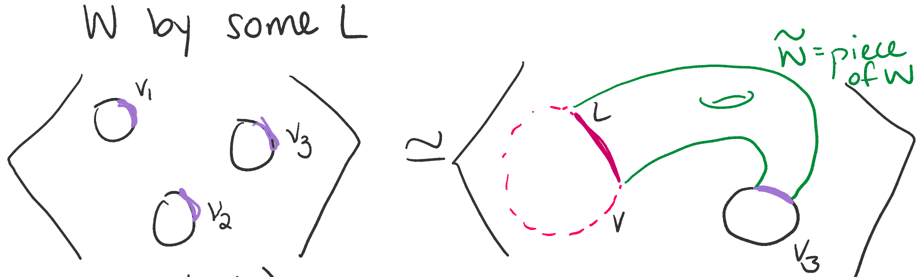
abelian group w/generator:

generator for $S_0^2(X)$:

1) choose a collection of disjoint balls $B_i^4 \subset X$
2) choose a link $L_i \subset \partial B_i^4 = S^3$
3) W = smooth surface in X, $\partial W = \bigsqcup L_i$ "lasagne sheets"
4) Choose some classes $v_i \in Kh(L_i)$ $\forall i$

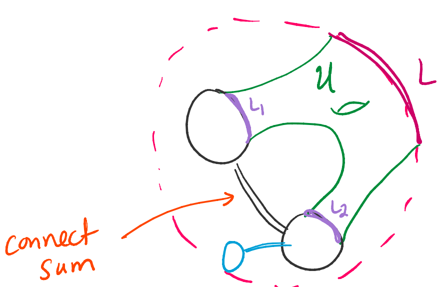
relations:

1) Choose a big B^4 containing several small B_i^4 & intersecting W by some L



$v_i \in Kh(L_i)$

where $v = \Phi(v_1 \otimes v_2)$

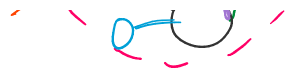


$L_1 \sqcup L_2$ in $S^3 \# S^3 = S^3$

$\mathcal{U}$ = surface connecting $L_1 \cup L_2$ & L_3 in $S^3 \times [0,1]$

$\Rightarrow$ defines a map

connect sum



$\Rightarrow$ defines a map

$$Kh(L_1 \sqcup L_2) = Kh(L_1) \otimes Kh(L_2) \to Kh(L_3)$$

Remark: Does not matter how we connectsum

Goals:

1) Can we compute this for some X?
   - Manolescu, Neithalath: yes, X has only 2-handles

     $\updownarrow$

     Kh(cables of specific links $\approx$ cores of handles)
   - Manolescu, Walker, Weidrich : 1-, 3-handles

     $\hookrightarrow$ Paul come in March? (talk)

$\to$ gl_2 Khovanov-Rozansky is this

$\to$ triply graded homology doesn't work nice topologically must start w/ braid

Hope: · Interesting invariant 4-mflds

· detecting exotic smooth structures?

"Manolescu-Neithalath"

Next time $\to$ §3: Cabled Khovanov homology

§4: Thm: For 2-handlebody X

Cabled Kh $\simeq S_0^2(X)$