MAT 145, Spring 2020

Practice problems for Midterm 1

1. Find the number of the following subsets of $\{1, \ldots, 7\}$:
(a) all 3-element subsets
(b) 3-element subsets containing 1
(c) 3-element subsets containing both 1 and 2
(d) 3-element subsets containing 1 and not containing 2.

2. There are 40000 students in UC Davis. Prove that there are at least 100 of them which share the same birthday.

3. Find the number of 4-tuples (a, b, c, d) of non-negative integers such that $a + b + c + d = 17$.

4. In some first grade class there are 12 boys and 12 girls. The teacher needs to place them at 4 tables such that there are 3 girls and 3 boys at each table. In how many ways could the teacher achieve this?

5. Use induction to prove that

$$1 + 3 + 9 + \ldots + 3^n = \frac{1}{2}(3^{n+1} - 1).$$

6. Find the number of anagrams for the word LOLLIPOP.

7. Find a general formula for the sum $n + (n+1) + \ldots + 2n$ and prove it for all n using induction.

8. Find the coefficient at x^8y^3 in the expansion of $(x + y)^{11}$.

9. Students take four classes A, B, C, D which run at the following times: A from 12:00 to 1:00, B from 12:30 to 1:30, C from 1:00 to 2:00 and D from 1:30 to 2:30. A student cannot register for two classes which overlap. Each of four classes has 100 registered students, 20 students take both A and C, 30 students take both B and D and 40 students take both A and D. How many students are there in total?

10. Solve the equation $\binom{n}{3} = 5n$.