MAT 145, Spring 2020
Practice problems for Midterm 2

1. Let us extend Fibonacci numbers to negative direction such that the recursion $F_{n+1} = F_n + F_{n-1}$ still holds. We get a sequence

$$...5, -3, 2, -1, 1, 0, 1, 1, 2, 3, ...$$

where $F_{-5} = 5, F_{-4} = -3, F_{-3} = 2, F_{-2} = -1, F_{-1} = 1, F_0 = 0$ and so on. Prove that $F_{-n} = (-1)^{n+1} F_n$ for all n.

2*. Are there Fibonacci numbers divisible by 10? Find one or prove they do not exist.

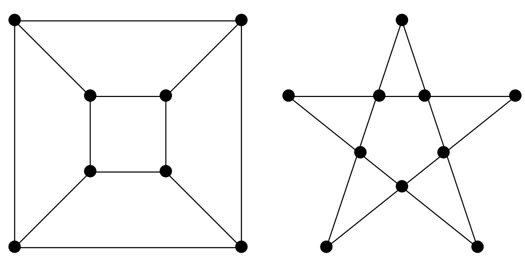
3. Prove that Fibonacci numbers satisfy the identity

$$F_1 + 2F_2 + \ldots + nF_n = nF_{n+2} - F_{n+3} + 2$$

4. In a rooted tree every vertex is either a leaf or has exactly 3 children. There are 11 leaves. How many vertices are there in this graph?

5. Is there a graph with (a) four (b) five (c) six vertices such that the degree of each vertex equals 3? Draw examples of such graphs or prove that they do not exist.

6. Are there Eulerian walks in the following graphs? Give an example or prove that they do not exist.



7. Can one color the vertices of graphs from Problem 6 in black and white so that no two verices of the same color are connected by an edge?

8. Give an example of a spanning tree for graphs in Problem 6. You do not need to find all spanning trees.

9 In some tournament 15 teams are playing with each other in some order. Prove that at every moment there is a team which played even number of games.

10. Two opposite corner squares are cut out from the chessboard (see picture). Is it possible to cover the remaining piece with 1×2 dominoes? *Hint: use chessboard coloring.*