MAT 145, Spring 2020
Due before 12:10 on Monday, April 6

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. (20 points) What is the number of positive integers with (a) at most n decimal digits (b) exactly n digits?

2. (20 points) Prove by induction that $1 + 2 + \ldots + n = \frac{n(n+1)}{2}$.

3. (20 points) Prove by induction that

$$1 \cdot 2 + 2 \cdot 3 + 3 \cdot 4 + \ldots + (n-1) \cdot n = \frac{(n-1)n(n+1)}{3}.$$

4. (20 points) Prove by induction that
$$1 + 2 + \ldots + 2^n = 2^{n+1} - 1.$$

5. (20 points) A drawer contains 6 pairs of black, 5 pairs of white, 5 pairs of red and 4 pairs of green socks.
(a) How many single socks do we have to take out to make sure that we take two socks with the same color?
(b) How many single socks do we have to take out to make sure that we take two socks with different colors?