

MAT 145, Spring 2020
Homework 2

Due before 12:10 on Monday, April 13

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. (20 points) How many numbers between 1 and 2020 are divisible by 3, 7 or 11?
2. (20 points) Prove that

$$\binom{n}{k} = \binom{n-2}{k} + 2\binom{n-2}{k-1} + \binom{n-2}{k-2}.$$

3. (20 points) Find the value of the following sum:

$$0 \cdot \binom{n}{0} + 1 \cdot \binom{n}{1} + 2 \cdot \binom{n}{2} + \dots + n \cdot \binom{n}{n}$$

for all n . Compute the values for small n first, guess the general formula and then prove it for all n .

4. (20 points) The city has a rectangular grid of streets. 6 avenues go from north to south, and 30 streets go from east to west. How many ways are there to get from the corner of 1st street and 1st avenue to the corner of 30th street and 6th avenue?
5. (20 points) Prove the identity for all n :

$$1 + 2\binom{n}{1} + 4\binom{n}{2} + \dots + 2^n\binom{n}{n} = 3^n.$$