# MAT 145, Spring 2020
# Homework 4
**Due before 12:10 on Monday, April 27**

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

**1.** (20 points) Prove the identity for all $n \geq k \geq m$:

$$\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{n-m}{k-m}$$

**2.** (20 points) Use the result of Problem 1 above to prove the identity for all $n \geq m$:

$$\sum_{k=m}^{n}\binom{n}{k}\binom{k}{m} = \binom{n}{m}\binom{m}{m} + \ldots + \binom{n}{n}\binom{n}{m} = \binom{n}{m}2^{n-m}.$$

**3.** (20 points) Prove that the Fibonacci number $F_{3n}$ is even for all $n$.

**4.** (20 points) Prove the identity for Fibonacci numbers:

$$F_1 + F_3 + \ldots + F_{2n-1} = F_{2n}$$

**5.** (20 points) In how many ways can you cover a $2 \times n$ chessboard with dominoes ($2 \times 1$ rectangles)?