

MAT 145, Spring 2020  
Homework 5

Due before 12:10 on Monday, May 4

*Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.*

Problems 1-3 are about *Lucas numbers* defined by

$$L_1 = 1, L_2 = 3 \text{ and } L_{n+1} = L_n + L_{n-1}$$

1. (20 points) Prove that  $L_n = 2F_{n+1} - F_n$ .
2. (20 points) Prove that

$$L_n = \left( \frac{1 + \sqrt{5}}{2} \right)^n + \left( \frac{1 - \sqrt{5}}{2} \right)^n$$

3. (20 points) A round flower bed has  $n \geq 2$  slots numbered from 1 to  $n$ . Prove that there are  $L_n$  ways to plant tulips and lilies in such a way that there are no two lilies planted next to each other (but two tulips can be planted next to each other).
4. (20 points) Prove the identity  $F_{a+b+1} = F_{a+1}F_{b+1} + F_aF_b$ .
5. (20 points) Draw all graphs with 2, 3 and 4 vertices.