MAT 145, Spring 2020
Homework 7
Due before 12:10 on Wednesday, May 20

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. (20 points) Prove that a graph with n nodes and m edges ($m < n$) has at least $n - m$ connected components.

2. (20 points) Prove that if a tree has a node of degree d, then it has at least d leaves.

3. (20 points) A rooted binary tree has height n, that is, the distance from every vertex to the root is at most n. What is the maximal number of vertices in this tree?

4. (20 points) A tree with 9 vertices (labeled from 0 to 8) has second row in Prüfer code 03204450. Reconstruct the first row in Prüfer code and the tree.

5. (20 points) Prove that the vertices in a tree can be colored black and white such that no two vertices of the same color are connected by an edge.