MAT 145, Spring 2020
Homework 8
Due before 12:10 on Wednesday, June 3

Please write the homework solutions in connected sentences and explain your work. Mark the answers to each question. Scan or take pictures of your homework and upload it to Gradescope before due time.

1. (20 points) Consider the polyhedron obtained by truncating (cutting off) all vertices of a regular tetrahedron. How many vertices, edges and faces does it have? Draw the corresponding planar graph.

2. (20 points) A polyhedron has 12 faces, each has the shape of rhombus.

(a) How many edges does it have?

(b) How many vertices does it have? *Hint: use Euler formula*

3. (20 points) Prove that in a planar bipartite graph one has $e \geq 2f$. *Hint: prove that* $2e \geq 4f$

4. (20 points) The graph $K_{3,3}$ has 3 black vertices and 3 white vertices, and every black vertex is connected with every white vertex. Use Problem 3 to prove that $K_{3,3}$ is not planar.

5. (20 points) The seams on a soccer ball form a graph. It has 60 vertices, each of degree 3. All faces are either pentagons or hexagons.

(a) How many edges does it have?

(b) How many faces in total does it have?

(c) How many pentagons are among the faces?