MAT 146, Spring 2019
Practice problems for Midterm 1

Note that this practice sheet contains more problems than the actual midterm

1. Find the formula for the coefficients a_n of the series $A(z) = \sum_{n=0}^{\infty} a_n z^n$ where

(a) $A(z) = \frac{1}{1+z}$

(b) $A(z) = \frac{1}{(1-3z)^2}$

(c) $A(z) = \frac{2+z}{(1-2z)(1-3z)}$

2. Find the generating functions for the following sequences:

(a) $a_n = 3n + (-1)^n$

(b) $a_n = 7 \cdot 5^n - 5 \cdot 7^n$

(c) $a_n = n^2 + n + 1$

3. Given the following recurrences for a_n, compute the generating functions:

(a) $a_{n+1} = 3a_n + 1, a_0 = 0$

(b) $a_{n+1} = a_n + 5^n, a_0 = 0$

4. Use generating functions to solve the following recurrences:

(a) $a_{n+2} = 2a_{n+1} - a_n,\ a_0 = 1, a_1 = 3.$

(b) $a_{n+1} = 2a_n + 5^n, a_0 = 0.$

5*. Consider all sequences $(a_1, \ldots, a_n)$ such that a_i are nonnegative integers and $a_i + a_{i+1} \leq 2$. Let P_n, Q_n and R_n be the number of such sequences which start from $0, 1$ and 2 respectively.

(a) Compute P_n, Q_n, R_n by writing down all such sequences for $n = 1, 2, 3$.

(b) Prove that P_n, Q_n, R_n satisfy the recurrence relations:

$$P_n = P_{n-1} + Q_{n-1} + R_{n-1}$$
$$Q_n = P_{n-1} + Q_{n-1}$$
$$R_n = Q_{n-1}.$$

(c) Translate the above equations into linear equations for the generating functions for P_n, Q_n, R_n.

(d) Solve these equations and compute the generating functions for P_n, Q_n, R_n.