MAT 146, Spring 2019
Practice problems for Midterm 2

Note that this practice sheet contains more problems than the actual midterm

1. Find all exponential generating functions $A(x) = \sum a_n \frac{x^n}{n!}$ satisfying the differential equation (a)$A''(x) = -A(x)$ (b) $A''(x) = A(x)$.

2. Find a closed formula for the exponential generating function $A(x) = \sum a_n \frac{x^n}{n!}$ where $a_n = 3^n - n \cdot 2^n$.

3. Find a closed formula for the exponential generating function $A(x) = \sum a_n \frac{x^n}{n!}$ where a_n satisfy the recursion $a_{n+1} = (n+1)(a_n - n + 1)$, $a_0 = 1$.

4. Find the generating function for the number of labeled graphs where there are 1 or 2 edges at each vertex.

5. Find the generating function for the number of permutations with no cycle of length 7.

6. Find the number of permutations of 10 elements, with disjoint cycles of length 1,2,3 and 4.

7*. a) Prove that the generating function for the number of hands with *even* number of cards equals

$$\frac{e^{D(x)} + e^{-D(x)}}{2}.$$

Hint: use the general two-variable exponential formula

b) Find the generating function for the number of permutations with even number of cycles.